The Unity of Science in Early-Modern Philosophy: Subalternation, Metaphysics and the Geometrical Manner in Scholasticism, Galileo and Descartes

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The project of constructing a complete system of knowledge—a system capable of integrating all that is and could possibly be known—was common to many early-modern philosophers and was championed with particular alacrity by René Descartes. The inspiration for this project often came from mathematics in general and from geometry in particular: Just as propositions were ordered in a geometrical demonstration, the argument went, so should propositions be ordered in an overall system of knowledge. Science, it was thought, had to proceed *more geometrico*.

I offer a new interpretation of 'science *more geometrico*' based on an analysis of the explanatory forms used in certain branches of geometry. These branches were optics, astronomy, and mechanics; the so-called subalternate, subordinate, or mixed-mathematical sciences. In Part I, I investigate the nature of the mixed-mathematical sciences according to Aristotle and some 'liberal Jesuit' scholastic-Aristotelians. In Part II, I analyze the metaphysics and physics of Descartes' *Principles of Philosophy* (1644, 1647) in light of the findings of Part I and an example from Galileo. I conclude by arguing that we must broaden our understanding of the early-modern conception of 'science *more geometrico*' to include concepts taken from the mixed-mathematical sciences. These render the geometrical manner more flexible than previously thought.

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PREFACE

Much too often, this dissertation came close to never being written. The fact that it is now written is due to the presence and patience of several people.

First are my advisers, Ted McGuire and Peter Machamer. Ted McGuire's guidance in the early years of my education is responsible for the arc of my academic trajectory. He first introduced me to history as a way of practicing philosophy and to the seventeenth-century as particularly rich field of study. Moreover, throughout my years in Pittsburgh, Ted has repeatedly exemplified for me the true scholarly stance. His passion and kind-hearted criticism always energize and are matchless in our Hobbesian academic world. Many years will pass before I can inspire so effortlessly.

Peter Machamer has easily been the greatest influence on this dissertation. Without his advice, I would have never fixed on the subalternate sciences as a subject of study and without his insight, I would have never seen clear through the problems they pose. Peter's kind words are few but unquestionably genuine, and his kind actions are equally genuine yet innumerable. For all of them, I am grateful. Peter's example will also be lasting: in our academic world of *res cogitantes*, Peter philosophizes as a doer. Many years will pass before I can get to the heart of things half as fast.

Jim Lennox and Paolo Palmieri have been invaluable committee members. In the great loneliness of authorship, I heard their uncompromisingly sharp criticisms echoing around me more than those of any others. Perhaps without them I would have paused less while writing, but my writing would have undoubtedly suffered. Paolo deserves additional thanks for structuring some of his courses to my needs and tolerating my juvenile Latin. Jim deserves additional thanks for talking at length about Chapter 2 and sharing with me his own paper on the same subject.

Dan Garber's support and willingness to work with me despite a paucity of drafts has been truly unbelievable. I wish I had been more productive and more punctual, so that I could have worked with him more. As things stand, I can only thank him profusely. Apart from our actual interactions, it was Dan's work on Mersenne and Galileo that convinced me that the subject of mixed-mathematics in the early seventeenth-century is worth investigating, and I'd like to thank him, once again, for that.

Fellow brunchers Erik Anger, Brian Hepburn, and David Miller read very early drafts of these ideas and helped me get started with writing. Chris Smeenk, although not involved with this dissertation, has been a steady cheerful presence in my intellectual life. To all, thank you.

And finally, my eternal and unending gratitude goes to Kate, who smiles at my flights of fancy and bears me when I'm down. She is my true companion. I would have given up without her, a long time ago.

ABBREVIATIONS

Unless otherwise noted, the following translations and editions of primary sources are used.

Physics	Charlton, William (1970). *Aristotle's Physics. Books I & II.* Oxford: Clarendon Press.
Summa	Eustachius a Sancto Paulo (1609). *Summa philosophiae quadripartita, de rebus Dialecticis, Ethicis, Physicis, & Metaphysicis.* Paris.
Disputationes Metaphysicae	Suárez, Francisco (1597). *Disputationes Metaphysicae.* Salamanca. Edition of 1619 reproduced in *Opera Omnia* (vol. 25–26), Hildesheim: Olms, 1965. Text reproduced by Salvador Castellote, Jean-Paul Coujou and John P. Doyle at http://homepage.ruhr-uni-bochum.de/Michael.Renemann/suarez/index.html
Principles	Descartes, René (1983 [1644]). *Principles of Philosophy*, trans. V. R. Miller and R. P. Miller. Dordrecht: D. Reidel. Translation of *Principia philosophiae* of 1644 with additional material from the French translation of 1647.
Le Monde	Descartes, René (1998 [1632]). *The World and Other Writings*, ed. and trans. S. Gaukroger. Cambridge; New York: Cambridge University Press.
Regulae	Descartes, René (1998). *Regulae Ad Directionem Ingenii = Rules for the Direction of the Natural Intelligence: A Bilingual Edition of the Cartesian Treatise on Method*, ed. and trans. George Heffernan. Amsterdam; Rodopoi.
AT	Descartes, René (1964–1974). *Oeuvres de Descartes*, ed. Charles Adam and Paul Tannery, nouvelle présentation. Paris: J. Vrin. *If translations of AT texts available in the sources below, I use them. Items lower on the list are only used if translations are not available in higher items. If translation are not available at all, I quote the original language.*

CSM	Descartes, René (1985). *The Philosophical Writings of Descartes*, trans. John Cottingham, Robert Stoothoff, and Dugald Murdoch. 2 Volumes. Cambridge; New York: Cambridge University Press.
CSMK	Descartes, René (1991). *The Philosophical Writings of Descartes. Volume III: The Correspondence*, trans. John Cottingham, Robert Stoothoff, Dugald Murdoch, and Anthony Kenny. Cambridge; New York: Cambridge University Press.

1.0 INTRODUCTION

> If one reflects somewhat upon the connection that discoveries have with one another, it is readily apparent that the sciences and the arts are mutually supporting, and that consequently there is a chain that binds them together. But, if it is often difficult to reduce each particular science or art to a small number of rules or general notions, it is no less difficult to encompass the infinitely varied branches of human knowledge in a truly unified system.
>
> *Discours préliminaire*
> JEAN D'ALEMBERT

The project of constructing a unified system of knowledge—a system capable of integrating all that is and could possibly be known—was common to many early-modern philosophers. René Descartes, for example, after listing how his *Principles of Philosophy* (1644, 1647) might be further developed, pronounced with characteristic zeal:

> [This] is what I should have to do in order to give to mankind *a body of philosophy that is quite complete*; and I do not yet feel so old... that I would not now boldly try to bring the plan to its conclusion[.][1]

Descartes' confidence may seem a mark of naiveté to a modern philosopher, but it is also indicative of the courage and conviction of the philosophical enterprise in early-modern times. Unlike d'Alembert's cautious evaluations in the *Discours préliminaire* or Carnap's self-consciously preliminary investigations in *The Unity of Science*, early-modern philosophers often portrayed themselves as being unbearably close to, if not having already achieved, their unifying goal. This dissertation is concerned with the extent to which early-modern philosophers succeeded in their unifying goal, the tools that allowed them to do so, and how the perception of their efforts could have been seen by their philosophically educated con-

[1] AT IXB 17, CSM I 188, emphasis added. Descartes goes on to say that he lacks the funds required to carry out this plan and appeals to monetary aid from "the public", thereby filling out a seventeenth century equivalent of a grant application.

temporaries. It focuses on a particular structuring principle around which unified systems of knowledge were built; namely, the geometrical manner and the notion(s) of deduction implicit in it.

When undertaking this sort of project, an appropriate first step would be to establish the relevant existence claim: i.e., to demonstrate that *in fact* a sufficiently wide cross-section of early-modern philosophers sought after unified systems of knowledge. The evidence, however, is too prevalent to require independent citation. Descartes, Hobbes, and Spinoza, not to mention Bacon, Gassendi, and Mersenne each made well-known noises about formulating a 'system of philosophy'. In fact, in contemporary scholarship the line drawn between early-modern "philosophers" and "scientists" often depends on the extent to which the actors in question sought to bind their innovations into unified systems or were satisfied with formulating particular solutions to particular problems.[2] Moreover, although the distinction between philosophers and scientists is sometimes ahistorical, it seems to me fair to call those who sought after unified systems "philosophers" not because they sought systematicity in itself, but because they used distinctly *philosophical* tools to unify their systems, tools that in their own times would have been identified as the purview of the philosopher. This claim—that distinctly philosophical tools were used to unify systems of knowledge—will form the cornerstone of my overall argument regarding the geometrical manner. I will argue that those tools were taken from reflection on branches of mathematics, the so-called subalternate, subordinate or mixed-mathematical sciences.

In order to articulate this overall argument—(as I will do in §1.2)—I must first set the historical scene in §1.1. The remainder of this introduction will address some preliminaries. Apart from spelling out the main argument and considering preliminary issues, I'd like to show in this introduction that the set of topics studied in this dissertation dovetail with one another: If one attempts to explain the early-modern drive for unification, the concept of subalternation and the mixed mathematical sciences—both of which will be defined shortly—naturally suggest themselves as *loci* for study. Similarly, if one attempts to clarify the nature of (certain) early-modern mathematical thought about nature, one is lead to the concept of subalternation, the nature of the scholastic-Aristotelian classification of sciences, and the

[2] See, e.g., Garber 1992: 307–308, Gaukroger 1989: 104ff., and Hatfield 2001: 398.

nature of unified systems of knowledge *writ large*. These topics are embedded in a vast tapestry of early-modern philosophy and science, but I believe they constitute a single thread: pull on one topic, and you will end up with the rest.

Before any unweaving, I should make one detail clear: by "a unified system of knowledge" or just "a system of knowledge" I mean something other than "a systematic philosophy". The notions differ insofar as a philosophy can be systematic—i.e., have the proper logical connections between its parts and tackle a variety of topics in a roughly uniform manner—without imputing systematicity to the whole of human knowledge. This is easy to see in the case of a radical sceptic. The radical sceptic, e.g., Sextus Empiricus, might have a perfectly systematic theory of knowledge, and yet not even believe in the existence of human knowledge. Less radical cases, e.g., Richard Rorty in *Philosophy and the Mirror of Nature*, Michel Foucault in *Les mots et les choses*, or John Dupré in *The Disunity of Science*, might not have sceptical doubts of the same kind but still hold that human knowledge is essentially disjoint.

For this reason, although the philosophers studied in this dissertation—i.e., Descartes and some scholastic predecessors—are all systematic philosophers, I do not study them insofar as their philosophies were systematic, but insofar as according to those philosophies the whole of human knowledge constituted a system. And although there is no *philosophical* reason why the two types of production must coincide, there are several *historical* reasons why they in fact coincided in the early- and mid-seventeenth century.

1.1 THE UNITY OF SCIENCE IN EARLY-MODERNITY

There are several historical reasons why early-modern philosophers chose to pursue systematic visions of human knowledge. The first is that many sought to replace the comprehensive and tightly bound body of scholastic-Aristotelianism by a similarly comprehensive alternative.[3] Although scholastic-Aristotelianism was under heavy, multi-flanked attack at the

[3] At this point in a work on early-modern philosophy, one must justifiably genuflect before the first chapter of Schmitt 1983, "Renaissance Aristotelianisms". There, Charles Schmitt argues that Renaissance Aristotelianism was not a fixed way of philosophizing, but a growing number of different ways of fitting

time, its scope remained a benchmark for philosophical success. Scholastic-Aristotelianism in its many varieties encompassed all that was thought to be known—at least by its own lights—and it did so by way of highly articulated conceptual scheme which—again, at least by its own lights—mirrored the highly organized character of reality.[4] Many early-modern philosophers rejected the notion that human ideas can directly mirror reality, but they were nevertheless eager to replace scholastic-Aristotelianism with a system of equal reach. This eagerness was motivated in part by concrete concerns. Due to scholastic-Aristotelianism's comprehensiveness, its influence extended far beyond the realm of philosophical ideas to the constitution of teaching curricula, social institutions, and even theological doctrine. In a sense, scholastic-Aristotelianism had weaved itself into the very fabric of late medieval, Renaissance, and early-modern European life. Consequently, in order to replace it without unraveling early-modern culture, early-modern philosophers had to concoct a system that could serve the same constitutive roles. The very unity of early-modern societies relied in some measure on the substitution of scholastic-Aristotelianism with a system of similar scope and coherence.[5]

A second reason early-modern philosophers chose to pursue systematic visions of human knowledge is more directly intellectual. As is well known, early seventeenth century philosophical, political, and natural-scientific thought underwent a skeptical crisis.[6] As the new

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Aristotelian texts with new-found knowledge. Speaking of 'Renaissance Aristotelian*ism*', the argument goes, glosses over important differences between the various Renaissance Aristotelian*isms*. (See also Grant 1987, Nederman 1996.)

It seems to me, however, that while this point was crucial to the development of historical scholarship when it was penned, the scholarly community has since digested it sufficiently. We can now self-consciously return to speaking of "Renaissance Aristotelian*ism*", but—having read Schmitt— under suitably delimited contexts and, of course, having provided our due diligence. I believe that in the matter above, due diligence (e.g., in Des Chene 1996 and Ariew 1999) has shown that despite important differences, scholastic-Aristotelianisms shared fundamental features that merit speaking, in a focused manner, of "Renaissance Aristotelianism". In the remainder of this dissertation, I also intend to argue that regarding the subject of "subalternation" significant homogeneity existed.

[4]The reason for this is straightforward, at least least in its broad outlines. Reality, according to scholastic-Aristotelianism, was an inherently ordered whole. However, since human ideas were thought to be constituted by the same forms from which real existents were constituted (yet lacking their proper matter), it seemed that the structure of ideas in the human mind could mirror the inherently ordered structure of reality. Of course, there are countless subtle and not-so-subtle variations here, particularly within more nominalist versions of medieval aristotelianism. See, for example, Pasnau 1997, Schmaltz 1997 and Kretzmann et al. 1982: Ch. 30.

[5]There are an increasing number of studies on the cultural centrality of scholastic-Aristotelianism. The case is both summarized and strongly argued for in Garber 2002a.

[6]See the seminal Popkin 1964, as well as Curley 1978, Yolton 1975 and newer essays in Watson 1988.

sciences delivered strong evidence that the scholastic-Aristotelian view of nature was wildly off the mark and as shifting political and religious allegiances destabilized the European map and personal consciousness, the need for certainty and security grew.[7] Overcoming skepticism became a *desideratum* of any philosophy, and a comprehensive vision of human knowledge often a tool in its attainment. Comprehensiveness was a tool against skepticism because in a properly unified system of knowledge, skeptical arguments need only be answered once. A successful answer to them propagates through the system and wards off doubts about any subject with which the system is concerned. This use of epistemological foundationalism to combat scepticism became a staple of early-modern philosophy and in contemporary writing, particularly of an introductory sort, is often presented as it *raison d'être*.[8]

A third reason some early-modern philosophers chose to pursue systematic visions of human knowledge stems from the second and concerns the increasingly evident success of mathematics. As scholastic-Aristotelian science lost its standing, the branches of mathematics—geometry, and to a lesser extend arithmetic and algebra—provided working models for a successful and seemingly indubitable way of knowing.[9] Science, it was increasingly thought, had to be proceed *ordine* or *more geometrico*.[10] Thus, the geometrical manner inspired a method of achieving unity within systems of knowledge: Just as propositions were ordered in a geometrical demonstration, the argument went, so should propositions be ordered in an overall system of knowledge. Of course, what exactly this 'order' amounts to is (and was) an open question, but an orthodox claim is that its exemplars were the proofs found in the

[7]For an admirable synthesis of the societal changes in early-modern Europe and their impact on the search for secure knowledge, with particular emphasis on the Reformation and the rise of free trade, see Machamer 1998c. Machamer believes the main impact of such changes was the promotion of first-person epistemologies, see Machamer 2000.

[8]In fact, one is hard-pressed to open an introductory textbook on early modern philosophy without encountering a familiar story about the rise of scepticism and the early-modern foundationalist fight against it. More detailed work include Berr 1960, Brush 1966, Popkin 2003, Curley 1978, Broughton 2002, Kisner 2005.

[9]Although advances in algebra were significant in the late Renaissance and early seventeenth century, few practical sciences or arts were based in algebra and thus the model for mathematical success remained geometry. Curiously, Descartes is probably one of the few philosophers who had a deep understanding of the methods of Algebra and, as Ken Manders argues, algebraic reasoning had a profound impact on Descartes' formulation of his notion of philosophical method, at least in the *Regulae* (see Manders 1995, the encyclopedic first Part of Sasaki 2003 and references therein).

[10]See Duchesneau 1983, Weingartnet 1983, Vega 1994, Moorman 1943, Angelis 1964.

first book of Euclid's *Elements*. For example, Edward Strong writes in his classic *Procedures and Metaphysics*:

> The clarity and certainty of the study of Euclid affords an ideal of knowledge when compared with the complexity and problematic nature of opinion. Descartes and Spinoza both see in geometry the clue to the true method of setting the mind in order as a preparation for setting all else in order. (Strong 1936: 27)

Although Euclid is still believed to be the primary example for the geometrical manner, and justly so, Strong did not have the last word. Scholarship since Strong has increasingly emphasized that Euclid's *Elements* was not the only methodological model for science and scientific demonstrations. Equally important were examples from Apollonius and Archimedes, and through them the notion of "geometrical analysis".[11] Apart from actual examples, ideas concerning the geometrical manner were further based on fragmentary writings by ancient commentators. Pappus, for example, provided much fodder for the Renaissance theory of the demonstrative *regressus*—a model of demonstration that combined the synthetic method of Euclid's *Elements* with the analytic method of ancient demonstrations—with a single passage in his *Collectio*.[12] Even more influential, but farther from real examples in Geometry, was Galen's commentary on method in his *Ars Medica*. The attempt to respect the authority of these disparate sources gave rise to a multiplicity of interpretations of "the geometrical method", but despite them, the fundamental point stands: the geometrical manner was valued because it was perceived to be the method of geometrical demonstrations, and these were in turn valued because these were taken as exemplars for human knowledge in general. Thus, for many early-modern philosophers, the whole of human knowledge needed to resemble geometry.

[11] See, e.g., Randall 1940: 205ff.; Gilbert 1960; Mahoney 1968, Hintikka and Remes 1974, Knorr 1986, Nets 2000, Berggren and Van Brummelen 2000, Otte and Panza 2002; Sasaki 2003: Ch. 1. In the historical literature, many treatments of analysis fall within the larger scope of studies on the *regressus* methodology, e.g., Gilbert 1963, Schmitt 1969, Jardine 1976, Poppi 2004, South 2005. However, it seems to me that attempts to fix the meaning of the *regressus* have been abandoned, and the scholarly consensus is that the term was rather flexible.

[12] As telegraphic as Pappus was, his was called "the most elaborate Greek utterance on the subject" by Heath 1981. See Behboud 1994 for a careful analysis of the text.

1.2 THE CENTRAL ARGUMENT: THE UNITY OF SCIENCE, THE GEOMETRICAL MANNER AND SUBALTERNATION

We can now come to the main argument of this dissertation. This dissertation offers a set of examples and concepts by which to study the geometrical manner—examples and concepts that, to the best of my knowledge, have not been used for this purpose before. Given these, I argue that the notion of the "geometrical manner" must be expanded.[13] The argument relies on the fact that several 'impure' sciences existed under the banner of geometry. These 'impure' geometrical sciences—optics, astronomy, and mechanics; more on these in §1.5—counted as 'impure' branches of geometry because they utilized geometrical principles in arguments about the natural world. This weak conception of geometry lies at the root of my argument. Because the meaning of "geometry" was wider than 'pure geometry', I argue that we must seriously entertain the possibility that the meaning of 'science *more geometrico*' was correlatively wider. Put explicitly, I argue that when early-modern philosophers deduced science *more geometrico*, they not only proceeded according the models of deduction inspired by Euclid or Pappus, but according to those models *as well as* the model(s) inspired by the impure mathematical sciences. Although the flexibility of the geometrical manner has been argued for by many authors, I believe my original contribution is to point to a specific set of concepts—those associated with the non-pure mathematical sciences—in light of which we can understand one of the many senses of the term.[14]

In order to clarify this wider sense, I turn to the scholastic-Aristotelian textbook tradition.[15] Historical scholarship has increasingly shown that the intellectual context of early-

[13]See note 29 for a review of the literature.

[14]In general, the literature on the flexibility of the geometrical manner is divided into those who have shown that the understanding of "mathematics" (and related concepts) was broader in early-modernity than it is now (e.g., Duchesneau 1983, Guéroult 1953, Belaval 1960, Feldhay 1998, Dear 1995, and related work), and works cited in footnote 11, and those who have shown that the understanding of "deduction" (and related concepts) was similarly broad (e.g., for example, Clarke 1977, Garber 1978, Gaukroger 1989, Nadler 1990, Normore 1993, Recker 1993). This dissertation falls half-way between the two, and notes that with a certain way of doing mathematics about the natural world came a particular model of deduction.

[15]By turning to the scholastic-Aristotelian textbook tradition, the basic strategy of this dissertation is to contextualize the early-modern philosophical project. This approach is common for contemporary work and need not receive further justification here. Virtually all work in history of philosophy since the 1960s have shared this basic approach. In my own academic trajectory, the doors to this way of doing philosophy and history were opened by the pioneering work of J. E. McGuire. McGuire 1972 will always be one of my favorite essays, but there are many, many others.

modern philosophy was in large part formed by the scholastic-Aristotelian commentary and *cursus* tradition. Since scholastic-Aristotelianism provided much of the teaching material and doctrines for Europe's universities, it was a tradition on which nearly all educated persons were nurtured. In a way, scholastic-Aristotelian teaching materials, and by extension scholastic-Aristotelianism itself, provided the *lingua franca* of early-modern learned culture.[16] In the body of the dissertation, I use Eustachius a Sancto Paolo and Francisco Suárez as representatives of a particular 'liberal Jesuit' strain of this tradition. However, I rely on a less studied feature of the textbook tradition; namely, its repeated and systematic use of the notion of 'subalternation' in the context of disciplinary classification schemes—schemes according to which the sciences were ordered.[17]

Subalternation is important because the relationship between the non-pure geometrical sciences and pure, Euclidean geometry was articulated by means of this notion. Explicating it will be the task of later chapters, but for the time being suffice it to say that subalternation is an asymmetrical, binary, and transitive relation that concerns the way in which certain sciences—or more generally, branches of knowledge—depend on one another. Optics, for example, was generally said to be subalternate to geometry because optical proofs depended on geometrical principles. Geometry was *sine qua non* for optics in a way that optics was not for geometry.[18] With the meaning of subalternation in the scholastic-Aristotelian tradition articulated, I will turn to see whether the relation of "subalternation" helps shed light on the efforts of certain early-modern philosophers to construct science *more geometrico*. I'll use Descartes as example and argue that his system of knowledge (as articulated in the *Principles*) fits this broader model of the geometrical manner. That is, I argue that for those

[16]For studies of early-modern education, see Brockliss 2006, Grendler 2002, Feingold and Navarro-Brotons 2006, Lines 2001, Porter 1996, Schmitt 1984b, Murdoch and Sylla 1975, Mallet 1924, Consentino 1999, Howell 1961. More on this in §1.4.

[17]Work on disciplinary classification schemes is usually focused on the ancient and medieval periods, e.g., Weisheipl 1965, 1978, McKirahan 1978, Laird 1987, Livesey 1990, Livesey and Lombard 1989, Livesey and de Carlenis 1994, Steneck 1975. Philosophical studies on the Renaissance and early-modernity are often focused on the status of the mechanical sciences and arts, e.g., Machamer 1978, Lennox 1986, Laird 1986, Mikkeli 2001, Garber 2002b, Hattab 2005.

[18]Optional insert: In this introductory stage, what is most noteworthy about subalternation is that it is a concept most properly applied to whole sciences. Neither propositions, arguments, proof, methods or goals of sciences are subalternate to one another, but one science is subalternate to another. Sciences may be subalternated *by virtue* of certain propositions, arguments, or other parts of the science, but sciences themselves provide the *fundamenta* of the subalternation relation. Use this fact, to draw the subject back to unified systems of knowledge.

versed in scholastic-Aristotelianism, he would be understood as using "subalternation" in order to provide unity to disparate branches of knowledge within his overall system. Before approaching Descartes, I will also use the first day of Galileo's *Discorsi* as an example with which to flesh out my account of the mixed-mathematical sciences and the notion of subalternation implicit in them.

Before turning to the body of the work, I should attend to some preliminaries. First (§1.3), I offer another reason why focusing on the mixed-mathematical sciences and the notion of 'subalternation' might be a fruitful endeavor. This reason has to do with one of the great achievement of the early-modern period, that of bridging the divide between Mathematics and Physics and constructing a mathematical study of nature. Second (§1.4), I delimit the extension of what I've called the "textbook tradition" of scholastic-Aristotelianism, as well as the several traditions of mixed-mathematics. Third (§1.5), I defend the claim that paying attention to mixed-mathematics can revel something about early-modern philosophical system that has not been previously reveled by paying attention to mathematics *simpliciter*.

1.3 THE BIRTH OF MATHEMATICAL PHYSICS

Although the goal of this dissertation is to focus on the overall structure of early-modern systems of knowledge, one need not be interested in this topic *per se* in order to be driven to study the range of topics I have mentioned thus far. In fact, a more common concern in scholarship of the early-modern period—a concern with the rise of mathematical physics and the bridging of mathematics and physics—recommends the very same set of issues.

That many early-modern philosophers advocated joining mathematics—geometry in particular—and physics in a mathematical study of nature is clear. In the *Principles*, for example, Descartes wrote:

> The only principles which I accept, or require, in physics [*physica*] are those of geometry and abstract mathematics; these principles explain all natural phenomena[.] (*Principles* II. §64)

Hobbes was equally explicit. In *De homine*, he wrote:

> So physics (genuine physics, I mean) is usually included among the 'mixed mathematical sciences,' since it depends on geometry. (*Works* II 93, §10.5)

According to the mainstream scholastic-Aristotelian view, however, natural philosophy was inherently non-mathematical. *Physica* and mathematics—as disciplines—occupied different positions within nearly all variants of the scholastic-Aristotelian disciplinary classification scheme and this meant that their subject matters, principles, and methods were distinct.[19] The two disciplines, simply put, had little to do with one another. For example, according to basic tenets of scholastic metaphysics, the objects of *physica* and mathematics were intrinsically different. *Physica* was about the essences or natures of natural things, embodied forms that were non-separable from their material embodiment; mathematics was about the proper attributes of bodies, features that, although also embodied, could be conceived of independently of their material embodiment without thereby changing their character.[20] The first was concerned with the essential sources of change; the second with the essentially changeless. Consequently, in order to alter the relationship of *physica* and mathematics, one had to address their supposed metaphysical divide. In general, in order to bridge the gulf between *physica* and mathematics, early-modern philosophers had to change not only mathematics and physics themselves, but also the character of the classification scheme that kept them apart. Descartes' program for the reorientation of natural philosophy was expansive in precisely this way: it reached beyond the particularities of either *physica* and mathematics and impinged on the scholastic-Aristotelian conception of knowledge, its nature and organization, as a whole.

Of course, early-modern philosophers were not the first to advocate the mathematical study of nature. From antiquity, mathematics was recognized as a hallmark of human knowledge and a small set of natural phenomena were thought to be properly understood by mathematical reasoning. What was unique to the early-modern period was the notion that progressively larger, perhaps limitless, swathes of nature could submit to mathematical

[19] There were exceptions, of course. The only ones I am aware of are the classification schemes of Christoph Clavius, Roger Grosseteste, and Roger Bacon. I limit myself to discussing scholastic-Aristotelian schemes, not those that are more at home in the humanist tradition. Sources on humanist classification schemes are, e.g., Kelley 1997, Grafton and Siraisi 1999, Field and James 1993, Westman 1980b, Mahoney 1976, Howell 1961, Laird 1991, Ong 1983.

[20] I use 'proper attributes,' following the usual translation of Aristotle's *Metaphysics* 1078a16. In medieval parlance, 'proper attributes' were also known as 'properties.' See Henry 1982.

analysis. In antiquity and the middle-ages, such swathes were thought to be limited to the domains of Astronomy, Optics, Harmonics, and Mechanics.[21] The rest of nature was thought to be essentially non-mathematical. Nature's recalcitrance to mathematical treatment was attributed to its inherently changing and teleological character, not to a failure of human beings to describe it properly. Mathematics, it was thought, was simply not appropriate for analyzing (most of) nature. Despite this, the existence of the properly mathematical natural sciences—the so-called "mixed mathematical," "middle," or "subalternate" sciences—makes clear that the mathematical study of nature was not an enterprise born in the early-modern period. No doubt, the early modern impulse for mathematization was in certain ways novel and in virtually all ways broader than the impulse of earlier ages, but it was not entirely new. Consequently, it seems only natural that in order to understand the early-modern mathematization of nature, i.e., in order to understand how early-moderns bridged the gulf between *physica* and mathematics, we ought to examine it in light of earlier attempts at mathematization, i.e., in light of the traditions of the subalternate sciences.[22] In this way, when we begin by considering the birth of mathematical physics, the early-modern context draws us to consider the the way in which early-modern philosophers grappled with disciplinary classification schemes. This, of course, is the same set of topics I argued ought to be studied together in §1.2, but this line of reasoning gives us independent reason to think that they, in fact, ought to be studied together.

1.4 PRELIMINARY I: DELIMITING TRADITIONS

In the above sections, I use the plural "traditions" to refer to mixed-mathematics because there was no monolithic tradition of mixed-mathematics. It is now time to clarify the extension of the term. In actuality, relatively independent optical, harmonical, astronomical, and mechanical (also known as 'the science of weights') research programs flourished from antiq-

[21] For general treatments of mathematics in medieval and ancient sciences, see Grant and Murdoch 1987, Lindberg 1978a, Moody and Clagett 1952, Clagett 1948. For ancient mathematics see Fowler 1999, Knorr 1986, Mueller 1981, Netz 1999.

[22] Of course, I am not the first to claim this; see Machamer 1978, Lennox 1986, Garber 2004.

uity to the early-modern period. This fact is widely recognized in the historical literature and I take it for granted.

In addition, however, the overall tradition of mixed-mathematics can be divided along yet another dimension, into *practical* mixed-mathematics and *philosophical, textbook commentary* on mixed-mathematics. This distinction is crucial to my argument. Historiographically speaking, much of the confusion regarding the importance of the subalternate sciences for modern philosophy arises from a disregard for this division; i.e., from a conflation of actual mathematical practice and philosophical reflections on it. However, once these are distinguished, their distinct contributions to the thought of early-modern philosophers can be better understood.[23] I now offer two short characterizations of the practical and commentary traditions.

By "the practical tradition of mixed-mathematics", I mean the tradition of mathematical problem solving that extended, in the case of mechanics, from the pseudo-Aristotelian *Mechanica* to the work of Hero, Archimedes, and Jordanus to the mechanical treatises of Tartaglia, Benedetti, and Galileo. In the case of optics, the practical tradition extended from the work of Euclid and Ptolemy to Grosseteste, Bacon, and Witelo to Kepler, Descartes, and Hobbes. Each tradition was defined by a shared, albeit heterogeneous and evolving, set of techniques for solving problems (concerning, say, constrained motion or perspective) and shared texts to which practitioners responded. Within the mechanical tradition in particular, texts often lacked philosophical coherence. Practitioners of mechanics arrived at concrete and often piecemeal solutions to particular problems (say, why balances of different sizes are not equally sensitive) and did not attempt to form systematic theories to account for the overall reliability or truth of their solutions. Because of this philosophical incoherence, philosophers often misrepresented the practice of mechanics. In particular, they often simplified the various ways in which mathematical and physical considerations came together in actual mechanical reasoning. Optical texts, particularly after the introduction of Ibn al-

[23] Of course, the philosophical commentary tradition may be differently related to different strands within the practical tradition. For instance, since optics inspired much medieval thinking about the subalternate sciences (as well as many ocular metaphors for knowledge, the transmission of information from object to subject, etc.), the optical tradition may be more intimately related to the commentary tradition than the mechanical tradition. The development of these relations, however, is outside the scope of this project and has been already studied heavily. The essays collected in Crombie 1990 are particularly useful, as is the broader Lindberg 1976.

Haytham to the Latin west, were more philosophically coherent, but the textbook tradition still misrepresented the complexity of actual reasoning in them.[24] Generally speaking, although there was a brief period of intense interaction between traditions in the thirteenth century, by the Renaissance the practice of mixed-mathematics was largely independent of philosophical commentary on it. The two were practiced by different populations and defined by different concerns.[25]

By "the textbook commentary tradition," I mean the set of writings in metaphysics and epistemology/psychology—mostly textbook commentaries on Aristotle's *Posterior Analytics*, *Physics* II, *Metaphysics E & M*, and *De Anima*—that placed the knowledge produced by the practical traditions within a larger theoretical framework.[26] This tradition extended from the works of Aristotle, Philoponus and Simplicius to those of Roger Grosseteste, Roger Bacon and St. Thomas Aquinas, to those of Francisco Suárez, Jacopo Zabarella and their lesser contemporaries Eustchius a Sancto Paolo, Abra de Raconis, and John of St. Thomas. The central problem in this tradition—insofar as mixed-mathematics was concerned—was to articulate the nature of mathematical knowledge about the natural world and its relation to

[24] Despite the influence of al-Haytham, attempts to place optics in a comprehensive physical, physiological, mathematical, and philosophical context were rare. As David Lindberg notes, "the majority of scholars working in any areas of what we now call optics continued to regard the mathematics, physics (sometimes accompanied by psychology or epistemology), and physiology of light and vision as distinct enterprises and preferred to practice one or another of them in isolation from the rest. In short, the old disciplinary boundary lines continued to exercise a strong influence on scholarship, and for most of the Middle Ages the perspectivists (the practitioners of *perspectiva*) struggled on behalf of a losing cause" Lindberg 1978b: 338.

[25] Although certain Jesuit mathematicians (like Clavius) both practiced and expressed philosophical interest in mixed-mathematics, they were rare and a wide difference of opinion existed between them and lion's share of philosophers. See Feldhay 1998 and
Dear 1987.

[26] A distinction emerged in the late fifteenth-century between the *commentary* and the *cursus* form of the textbook. In a commentary, the order of exposition generally followed the order of the primary text commented upon (whether it was reproduced or not) and conformed to the *disputatio* style. In a *cursus*, the explicit *disputatio* and its strict back and forth of contrasting opinions was generally abandoned, but a series of comments on a given *quaestio* still framed discussion. Most significantly, however, in a *cursus* the order of exposition no longer mirrored that found in Aristotle's primary texts. Rather, topics followed a logical or thematic order. In a *cursus*, the original text had been almost entirely forgotten.

The distinction between the two forms of textbook is not crucial for our purposes, since both continued to be used in university education. Charles Schmitt, for example, notes that: "[T]he commentary form continued to be the basis of instruction in Italy and Spain longer than elsewhere. In northern Europe, the indication is that either a direct reading of the text or a cursus based on the Aristotelian corpus, but often arranged and tailored to meet specific needs, were in more common use than were commentaries... Still, commentaries continued to play a major, and perhaps dominant role in philosophical instruction during the whole of the sixteenth and much of the seventeenth century" (Schmitt 1983: 50). See also Schmitt et al. 1988: Ch. 23, Reif 1969, Brockliss 1987, Schmitt 1988, Grafton 1981.

other forms of knowledge, particularly purely mathematical and purely physical knowledge. Works in this tradition sought to explain in what manner mathematics about the physical world unified both physical and mathematical forms of reasoning, what its objects were, and what methods it used to study those objects.

The distinction between the practical and textbook traditions is important *at the very least* because early-modern reactions to them differed. A recent series of articles concerning Galileo's relationship to the subalternate sciences demonstrates how this fact can be easily forgotten. On the surface of things, there is a significant disagreement between Machamer 1978, Lennox 1986 and Laird 1997: Machamer 1978 and Lennox 1986 argue that Galileo's work should be understood as an extension of the tradition of the subalternate sciences, while Laird 1997 argues that it should not. However, the authors' disagreement arises because they understand 'the tradition of the subalternate sciences' differently. A closer look reveals that while Machamer and Lennox argue that Galileo's *actual mathematical reasoning* fit the mold of demonstration in the subalternate sciences, Laird argues that Galileo was not engaged with (and could have only been discouraged by!) any traditional *philosophical questions* regarding the subalternate sciences. The superficial disagreement of the authors is due to the fact that the tradition of the subalternate sciences can be understood as a philosophical one, concerned with the justification of certain modes of reasoning (as Laird understands it), or it can be understood as a practical one, concerned with the application of such reasoning to particular physical problems (as Machamer and Lennox understand it).[27] If this distinction is kept in mind, the above positions can be made compatible. Following Machamer and Lennox, we can say that Galileo's actual mathematical reasoning matched exemplars in the subalternate sciences. At the same time, we can heed Laird's warning and not forget that Galileo did not directly engage the philosophical issues surrounding those sciences.[28] The positions are only incompatible if we expect Galileo to have a uniform reaction to all aspects of the subalternate sciences. Distinguishing the practical and textbook traditions in this way is crucial for my

[27]Lennox, however, wishes to use Galileo's practice as evidence that he was more attuned to philosophical problems. In this sense, he lies somewhere between Machamer and Laird, but still focuses mostly on Galileo's practice.

[28]Wallace 1992 argues that Galileo actual reasoning followed exemplars in the subalternate sciences because Galileo was an avid student of the theory of demonstration of the *Posterior Analytics*. Consequently, in Wallace's work the distinction between the practical and textbook traditions of mixed-mathematics is often blurred. Laird's main argument is against Wallace and I am inclined to accept his conclusions.

argument because it allows me to treat scholastic commentary on mixed mathematics as a coherent and complete data-set. That is, it allows me to study the concept of subalternation *in textbooks and commentaries only*—without needing to account for its use in the actual practice of early-modern scientists. As noted above, the main justification for this is the *de facto* separation of mathematical practitioners and philosophical commentators in early-modernity.

1.5 PRELIMINARY II: DEFENDING MIXED-MATHEMATICS AS A SUBJECT OF STUDY

So far so good, but, historiographically speaking, why should we think that paying attention to theories about *mixed*-mathematics—as enshrined in scholastic-Aristotelian textbooks—can reveal something *novel* about the rise of mathematical physics or the character of early-modern philosophical systems? Why, that is, should we think that the textbook commentaries on mixed-mathematics can teach us something we've failed to learn from the vast remainder of philosophical literature, or from the many works that focus on the practice and theory of mathematics *simpliciter*?[29] The answer here involves both sociocultural and intellectual dimensions. First, although "subalternation" as a historiographical category may not shed much light on the rapidly evolving practice of the new sciences, it allows us to trace their *philosophical reception*. The content of the new sciences, it has long been recognized, matured primarily in the artisan classes of engineers, gunners, ship-builders, fortifiers, instrument-makers, and navigators—'mathematical practitioners', as they are often called.

[29] Studies that stress mixed-mathematics are a fairly recent addition to the historiographical spectrum. They begin with Gagné 1969, Ribeiro do Nascimento 1973, McMullin 1978, Livesey 1982 and continue to Lennox 1986, Meli 1992, Dear 1995, Garber 2000. Classic accounts of the rise of mathematics include Koyré 1943, Burtt 1954, Crombie 1959, Dijksterhius 1961, Hall 1966, Hall 1954, Butterfield 1951, Gillispie 1960. Newer accounts such as Mancuso 1997, Mahoney 1984, Westman 1980b, and Palmieri 2001 take a more contextualized approach, and have increased our understanding by paying closer attention to mathematical practice and mathematical practitioners. The rise of mathematics has also been related to larger socioeconomic, cultural, and technological issues, particularly the rise of capitalism and new social classes, the modern conception of the self, and the flourishing use of instrumentation; see, for examples , the classics Weber 1958 and Merton 1970, as well as the newer Shapin and Schaffer 1985, Daston and Vidal 2003.

These classes made little contact with the philosophy of their time.[30] As others have already argued, however, the new physical-*cum*-mathematical sciences were perceived as instances of mixed-mathematics—that is, subalternated sciences—by the philosophical *savants*, particularly in mid-seventeenth-century Paris, and their acceptance within this intellectual class is well worth paying attention to.[31] After all, Newton's *Mathematical Principles of Natural Philosophy*—the crowning achievement of the scientific revolution—was written by a named chair in a university, the bastion of the philosophical establishment! Thus, even if philosophical doctrines concerning mixed-mathematics had little impact on the technical and technological development of modern science,[32] their acceptance into the orthodox institutional infrastructure cannot be understood without accounting for how they were viewed by the philosophically minded.[33] The distinction between the practical traditions of mixed-mathematics and the philosophical commentary on them is meant to facilitate this type of accounting.

Second, there is a more philosophical reason for focusing on mixed-mathematics, but it concerns the rise of mathematical physics more than the overall character of early-modern systems of knowledge. If the reconstitution of natural philosophy as a mathematical science is seen only thought the lens of pure mathematics, it seems—dare I say?—*too* profound. Paying attention to the mediating role of mixed-mathematics shows that the gulf between mathematical and physical reasoning—which were thought by many to be fundamentally independent—was not as unbridgeable as some historians have thought.[34] Seventeenth cen-

[30] The *locus classicus* is Zilsel 1942, see also Rossi and Nelson 1970.

[31] See Garber 2004 and Dear 1995: particularly Ch. 6.

[32] But see Meli 1992: esp. 25–26. Referring to a somewhat different process, J. A. Bennett has called this mutual interaction a "negotiation" of traditions. The term is equally useful here, but what is being negotiated is an older philosophical understanding of a practical art with a newer philosophical world-view. See Bennett 1986.

[33] I don't mean to suggest that the institutional infrastructure remained unchanged. In the seventeenth century, the center of scientific activity shifted, at least temporarily, from the universities to relatively independent scientific academies. However, universities retained their exclusive role in education and a significant role in promoting research (see Rossi 2001: Ch. 16). Their process of conversion to the "new sciences" was slow and requires more study.

[34] The 'great tradition' historians—to borrow a phrase from H. F. Cohen—thought the mathematization of nature was a 'revolution' in the strictest sense of the term: an overthrow of one intellectual regime and its complete replacement with another. That is, they could not see how the separation of *physica* and mathematics could have been overcome by anything but an absolute rejection of scholastic-Aristotelian philosophy (see Cohen 1994: Ch. 2). Nevertheless, the work of Randall, Schmitt, Lohr and others has shown that there was as much continuity in the scientific revolution as there was revolution (see, e.g., Randall 1961

tury philosophers *even within the scholastic-Aristotelian tradition* were able to accept a good deal of mathematical reasoning about nature as long as it was sufficiently distinguished from genuine *physica*.[35] Keeping disciplinary boundaries intact allowed traditional philosophers to look benevolently, if condescendingly, upon the new scientists. However, it also provided the new scientists with the autonomy to develop unorthodox views and ultimately question the scholastic-Aristotelian boundaries within which they were confined.[36] This process of questioning appropriated some of the ideas that were initially used to keep mixed-mathematics in its proper place, particularly ones about the relationship of mixed-mathematics to mathematics and physics. These ideas served as middle ground for the intellectual orthodoxy and the new scientists and made the arguments of one group intelligible to the other.[37] Particularly within Mersenne's circle, this strategy was promoted because it allowed for the advocacy of the new sciences without any destabilizing effect on those political institutions (such as the universities) that were based on the promotion of the scholastic-Aristotelian worldview.[38] Focusing on the structure of the sciences is important precisely because such a strategy was undertaken in response to the strictures regarding the individuation and classification of sciences.

1.6 PLAN OF THE WORK

To sum, in this dissertation I offer a new interpretation of 'science *more geometrico*' based on an analysis of the explanatory forms used in certain branches of geometry. These branches were optics, astronomy, and mechanics; the so-called subalternate, subordinate, or mixed-mathematical sciences. In Part I, I investigate the nature of the mixed-mathematical sciences according to Aristotle and some 'liberal Jesuit' scholastic-Aristotelians. In Part II, the heart

and Schmitt 1984a). For the battle between the two historiographical approaches, see Eastwood 1991.

[35] See Dear 1995: Ch. 6, for example.

[36] See, e.g., as Meli 1992.

[37] Garber 2002b: 203 uses Peter Galison's metaphor of 'trading zones' to articulate this. Garber argues that the science of machines served as a trading zone for Aristotelians and the new scientists across which they could exchange ideas without a fuller commitment to dialogue and reconciliation. This trading zone was defined by disciplinary boundaries.

[38] The line of reasoning is based on two excellent articles: Garber 2002b: 203 and as Meli 1992. Garber discusses Mersenne's circle explicitly, Meli studies similar circumstances in late sixteenth-century Italy.

of the work, I analyze the metaphysics and physics of Descartes' *Principles of Philosophy* (1644, 1647) in light of the findings of Part I and an example from Galileo. I conclude by arguing that we must broaden our understanding of the early-modern conception of 'science *more geometrico*' to include concepts taken from the mixed-mathematical sciences. These render the geometrical manner more flexible than previously thought.

In Chapter 2, I show that according to Aristotle's account of mathematical abstraction and his theory of genera-crossing demonstrations, the mixed-mathematical sciences must be understood to treat matter and mathematical form *jointly*. This renders them unlike pure mathematics, which treats mathematical form as entirely divorced from matter, but also unlike natural science, which treats matter and form as intimately united. I call this the *tempered-hylomorphism* of mixed mathematics and use it to offer an interpretation of *Physics II.2*. I argue that Aristotle's discussion of mixed mathematics in the chapter is meant to ward off a likely misunderstanding by his Platonic audience regarding the interrelations of matter and form. Elements of this interpretation were recognized in antiquity, but have gone unnoticed by modern commentators.

In Chapter 3, I next show that the concept of *subalternation*—the idea that one science can 'borrow' features of another and apply them to its own domain—was used to analyze the nature of the mixed-mathematical sciences in later scholastic-Aristotelianism. I use Eustachius a Sancto Paolo's *Summa Philosophiae* (1609) and Francisco Suárez's *Disputationes Metaphysicae* (1597). I show that both thinkers also exported the concept of subalternation from the domain of mixed-mixed studied in Chapter 2 to an analysis of the structure of knowledge at large. In particular, both thinkers conceptualized the relation of *metaphysics* to the other sciences in terms of subalternation. Given the prevalence of these works, I use this as evidence for the claim that an educated person in the early seventeenth century would have understood the new sciences of the time in terms of subalternation and would have been familiar with the thesis that metaphysics subalternates the other sciences.

In Chapter 4, I show that *tempered-hylomorphism* and *subalternation* can be used to clarify the relationship between the first and second days of Galileo's *Two New Sciences* (1638), a relationship that has not been previously recognized in the secondary literature. This chapter aids in my analysis of Descartes by providing a working example of a mixed-mathematical demonstration and evidence for the claim that using mixed-mathematics as an analytical tool can bear fruit. The chapter also show that Galileo's first new science ought to be understood as the science of matter, a partner to his science of motion.

Following the elucidation of mixed-mathematical demonstration, in Chapter 5 I clarify the interrelation of metaphysics and physics in Descartes' *Principles of Philosophy.* I begin with a simple question: How can we demarcate Descartes' physics from his metaphysics given his repeated claims that the two constitute a continuous deductive whole *and* his de facto treatment of the two as distinct fields? After arguing that several interpretations of Descartes do not have the resources to answer this question, I offer an analysis based on the concept of subalternation. More precisely, I argue that the deduction of the laws of nature is an instance of subalternation; that is, that the laws are generated by taking metaphysical principles and applying them to the domain of matter in motion. I argue that reading Descartes' statements regarding the deduction of physics according to the model of subalternation allows us both to solve the demarcation problem and to respect Descartes' claims that metaphysics and physics form a deductive whole.

In conclusion, I argue that we must broaden our understanding of the early-modern conception of "science *more geometrico*". Since the mixed-mathematical sciences were thought to be branches of mathematics—mostly of geometry—the forms of demonstration used in them would have been understood by the philosophically educated as *geometrical forms of demonstration.* In other words, because the meaning of "geometry" was wider than "pure geometry", I argue that the meaning of "science *more geometrico*" was correlatively wider and shaped by the notions of *subalternation* and *tempered-hylomorphism.* The concept is therefore broader and more flexible than previously thought.

2.0 THE TEMPERED-HYLOMORPHISM OF MIXED-MATHEMATICS

This chapter aims at two separate but intertwined goals. First, it aims to expose the the basic concepts used in the Renaissance textbook tradition of commentary on mixed mathematics. These concepts do not define a particular position, but a landscape into which particular positions fit.[1] I pursue this goal by focusing on Aristotle's original texts. Of course, because Aristotle is separated from his Renaissance commentators by a wide temporal and conceptual gulf, no thesis stated of him can be true in the same way (with the same caveats, presuppositions, implications, etc.) of his commentators. However, because Aristotle draws the boundaries within which his commentators work, focusing on him is adequate for surveying the lay of the land. Consequently, in this chapter I allow myself to speak

[1] I am guilty of a historiographical sin. Although the phrase "mixed-mathematics" only came into existence in the seventeenth century, I use to it to refer to the entire tradition of commentary on Optics, Astronomy, Mechanics, and Harmonics. I believe this is excusable: since my focus is ultimately the seventeenth century, I use "mixed-mathematics" merely for uniformity of exposition. Since the terms "mixed-mathematics," "subalternate science" and "middle science" consistently referred to the same set of disciplines, I don't believe this should cause a problem. On the evolution of the term "mixed-mathematics," see Brown 1991.

A recent objection to this usage is made by Gaukroger and Schuster 2002: 535-538. They suggest that the terms 'mixed-mathematics', 'subalternate science, 'practical mathematics, and 'physico-mathematics' cannot be used interchangeably. Their argument is that each of the terms signifies a different way in which mathematical and physical reasoning were combined *in practice*, and that this manner of combination was precisely what was up for grabs in the early decades of the seventeenth century. Although I agree with their outlook, I believe that homogeneity cannot be found even within groups of practitioner who referred to their art using only one of the terms above. For example, both Kepler and Beeckman called their work 'physico-mathematics', yet Kepler believed that physical reasoning could invalidate mathematics whereas Beeckman did not (see AT X 52 and Miller 2008). Similarly, as Meli 1992 clearly shows, Benedetti and del Monte referred to their mechanical works as 'subalternate sciences', but held different views on the relation of physics to mathematics. Overall, I believe it is best to take the multiplicity of *practiced* approaches as irreducible and focus on the *philosophical* reconstruction of the relation of physics to mathematics. General divisions may emerge within the philosophical reconstructions, but the practice of the new science was, by its very nature, innovative and thus radically heterogeneous.

On another note, after urging terminological caution, Gaukroger and Schuster write that "[t]he term 'mixed-mathematics' had been framed *by Aristotle*..." (p. 537, emphasis added). They do not provide a textual reference. The claim, as far as I know, is false.

of "the scholastic-Aristotelian model of science", although the evidential basis is limited to Aristotle himself.[2]

This chapter's second aim is to articulate and defend the utility of the notion of *tempered-hylomorphism*: the idea that in scholastic-Aristotelianism a mixed-mathematical science treats material and mathematical considerations *jointly*, albeit in a way that falls short of the genuinely hylomorphic treatment demanded of a *natural* science by Aristotle. Although the notion of tempered-hylomorphism does not by itself recommend a specific way in which material and mathematical considerations ought to be treated jointly, I believe it is important to highlight the fact that it recommends the very idea of joint treatment, albeit of a circumscribed nature.[3] Tempered-hylomorphism will prove important for understanding the structure of Galileo's argument in the first two days of his *Discorsi*.

2.1 MIXED-MATHEMATICS: THE BASIC FRAMEWORK

Aristotle's formulation of the very notion of mixed-mathematics is constituted from numerous passages in a variety of texts. The relevant passages, as far as I know, are: *Posterior Analytics* I.9 75b8–9, 15; 76a10–12, 22–25; I.12 77b1–2, I.13 78b34–80a16; *Metaphysics* A.1 981a12–24, B.2 997b35–998a6, K.1 159b10–12, E.1 1025b25–28 (although the whole of E.1 is important), M.3, 1077b17–30, 1078a2–26 (although the whole of M is important); *Physics* 2.2 193b32–194a12; *De Anima* II.7, esp. 418b14ff; *Meteorlogica* III.2, 371b26–372a10, III.4; *De Partibus Animalium* I.1 639b7-10; *De Sensu* 437b11ff. In these works, Aristotle set out interlocking theories of scientific demonstration, disciplinary classification, and human cognition that allowed him to hold on to both horns of what seemed like an insoluble dilemma. On the one hand, he insisted that mathematics was inherently inadequate for explaining natural phenomena. On the other, he held that a mathematical understanding

[2] For the beginnings of a defense of such historiographically broad approaches, see Edwards 2007: 458.

[3] Although I will not argue for the following thesis in this dissertation, I believe the lack of specificity in the notion of tempered-hylomorphism makes it a useful tool for analyzing seventeenth-century mixed-mathematical sciences precisely because these often combined material and mathematical considerations in novel ways, as evidences by much of the literature on particular instances of mixed-mathematics; see Molland 1987, Lines 2006, Feldhay 2006, Gaukroger and Schuster 2002, Dear 1987, Laird 1986, Westman 1980a.

of a limited range of natural phenomena—the phenomena studied by what we now call the "subalternate," "middle" or "mixed" sciences—was nevertheless possible.[4] To understand the philosophical issues surrounding mixed-mathematics, we must begin with Aristotelian conception of science.

2.1.1 THE DEDUCTIVE STRUCTURE OF ARISTOTELIAN SCIENCE

On the Aristotelian model, a science (the Greek *episteme* or Latin *scientia*) is a deductively ordered chain of appropriately constructed syllogisms. In the first book of the *Posterior Analytics*, Aristotle set forth the requirements for constructing such *scientific* syllogisms, or *demonstrations*. Apart from obeying the formal rules governing all syllogisms, Aristotle demanded that the premises of scientific syllogisms be "true, primary, immediate [i.e., lacking a middle], better known than, prior to, and ground of [i.e., cause of]" its conclusions.[5] This list of requirements implies that when syllogisms are connected in deductive chains—i.e., when they form sciences—the premises of the top-most syllogisms must be more primary, more immediate, and better known than the propositions in the deductive chain below them. Such premises—the *principles* (archai) of a given syllogistic chain—are the grounds of the science in question; they reveal the true and ultimate causes of whatever subject the science is concerned with.[6] In effect, this model divides syllogisms into branches, with each branch comprised of the body of syllogisms that can be derived from some set of

[4] Later works on the philosophical framework of mixed-mathematics are mostly constituted by commentaries on these texts within the Aristotelian tradition; see, for example, Philoponus 1993, Simplicius 1997, Aquinas and Maurer 1963, Zabarella 1607, Coimbra [Collegium Conimbricensis] 1594, Suarez 78, Fonseca 1615, Toletus 1589.

The question of Plato's relationship to theories of mixed-mathematics is more complicated and will not be addressed here. However, as a first pass, I should note that although Plato championed the utility of mathematics more forcefully than Aristotle, he ought to take a back seat in any discussion of mixed-mathematics because he denied that mathematical entities could be perfectly instantiated in sensible, natural objects. In other words, he denied that natural objects *per se* could be mathematically (or more broadly, rationally) understood. According to the line of thinking inherent in theories of mixed-mathematics, however, mathematical understanding can be literally about objects in the sensible world. See, *Meno* 81a-86a, *Pheado* 73a-77a, *Euthydemus* 290b–c, *Pheado* 103–106, *Republic* 527a-d, and Annas 1976: Ch. 1 for Plato's views on the relationship between Forms, mathematical entities, and physical objects. It should also be noted that the *Timaeus*, a common text in the Renaissance, poses a problem for this clean separation of Platonic and Aristotelian positions. In the *Timaeus*, Plato holds that the intelligibility of the material world results from its constitution from archetypal geometrical shapes by the demiurgos.

[5] *Posterior Analytics*, I.2 71b20–22; translation by McKirahan 1992.

[6] See McKirahan 1992: Ch. 2 ,Weisheipl 1978; more general studies include Barnes et al. 1975, and Barnes 1969.

indemonstrable principles. Since such bodies of syllogisms constitute *sciences*, this model suggests what I'd like to call the *Principles Criterion* for individuating sciences. According to this criterion sciences are distinguished according to the sets of principles on which they are based. Conversely, sciences are considered a single or unified science if they share all their principles. Aristotle himself did not ultimately recommend this criterion (although he did believe that different sciences have different principles), but I believe it is useful for discussing the individuation of sciences. Importantly, this criterion rests on the idea that the premises of all but the top-most syllogisms in any given chain must be derived from other syllogisms in the same chain.

Interestingly, Aristotle and his followers allowed for certain exceptions to this scheme. They allowed, that is, for some demonstrations to borrow premises from deductive chains different than their own, for the premises of certain syllogisms in given sciences to be demonstrated in different sciences. In those cases, the former science was said to be "subalternated" to the latter. The term "subalternation" derives from the Latin translation of Aristotle's claim that such sciences stand "one beneath another," or "alterum sub altero."[7] Two pairs of sciences may serve as the principal examples of subalternation: optics and geometry, and harmonics and arithmetic. These sciences stood one beneath another because the lower ones—optics and harmonics—were allowed to borrow some of their premises from the higher ones—geometry and arithmetic. In other words, when one demonstrated propositions in optics, some of the premises from which these demonstration proceeded were geometrical. When one demonstrated propositions in harmonics, some of the premises were arithmetical. In general, all the mixed mathematical sciences borrowed premises (and by extension principles) from, or were subalternated to, the pure mathematical sciences.

The notion that certain sciences could borrow principles had some troubling consequences. Aristotle demanded that all terms in a scientific demonstration belong to the same subject-genus.[8] In the *Posterior Analytics*, the prohibition against intermingling genera was

[7] *Posterior Analytics* I.7 75b15. Although Aristotle's statements on subalternation focused on the relationship between the *principles* of a pair of sciences, later commentators expanded the notion to include the investigate methods used by those sciences; see Laird 1983, particularly on Aquinas and Aquinas and Maurer 1963. In contrast to Laird, Livesey 1982: x claims that Aristotle himself believed in the subalternation of methods.

[8] McKirahan 1992: Ch. 4 argues that that membership in a single genus is not an added condition, but results from the initial list of requirements listed above (in I.2 71b20ó22). This is not universally agreed

stated explicitly:

> It is not possible to demonstrate by crossing from another genus; for example, [it is not possible to prove] what is geometrical by means of arithmetic... [I]t is clearly impossible, for the extremes and the middles [i.e., the terms in the syllogism] must be from the same genus. (*Posterior Analytics* I.7 75a38-b10)

In the subalternate sciences, however, demonstration concerning one subject matter (say, optics) borrowed their principles from another subject matter (geometry). That is, because they used principles from one branch of knowledge to prove conclusion about another, they intermingled terms concerning one genus (physical entities) and terms concerning another (mathematical entities or mathematical properties) in the same syllogism. Of course, Aristotle was not blind to this possibility and stated the above prohibition against genera-crossing cautiously.[9] He wrote that "[in demonstrations] the genus must be either the same without qualification or *somehow the same* if the demonstration is going to cross [genera]" (emphasis added). In the case of the subalternate sciences, the subject-genera of demonstrations (physical entities and mathematical entities or properties) were not the same "without qualification." But how were they *somehow* the same?

2.1.2 THE GENERIC STRUCTURE OF PHYSICAL AND MATHEMATICAL OBJECTS

In order to understand this "somehow", we must first examine why Aristotle thought that natural and mathematical entities belonged to different genera "without qualification." One of Aristotle's clearest discussions is in *Metaphysics E*. There, Aristotle's task is to distinguish metaphysics from other branches of knowledge. His distinction rests on the idea that sciences are individuated according to their subject matter, or, in his words, that sciences "circumscribe some particular kind of being as an object of special concern" (1025b8–9).[10] I'd like to call this the *Object Criterion* for individuating sciences—sciences are individuated

upon, see Livesey 1982: 5–19.

[9] I have presented this in an order reverse from Aristotle's. Aristotle begins with the notion that certain syllogisms can cross genera and explains the notion of subalternation after. This should not make a difference.

[10] The translation is Richard Hope's. I am ignoring here the equivalence/conflict of two senses of 'metaphysics' that can be found in *Metaphysics E*. The first sense of metaphysics is of a science with some particular subject matter. The second sense is of a science lacking a particular subject matter. The distinction has been discussed by Jaeger 1948: 215–218, Livesey 1982: 38–46, and Frede 1987.

according to the type of object with which they are concerned. Aristotle states the criterion explicitly in *Posterior Analytics* I.28. In *Posterior Analytics* I.32, he argues that the Objects Criterion implies the Principles Criterion; i.e., that if two demonstrations concern different genera, then their principles cannot be the same. In the *Metaphysics E*, Aristotle argues that while metaphysics studies what is common to all being, other sciences study what is unique and essential to particular sorts of beings. In his own words, sciences "explain as best they can what pertains essentially to the kind of being their particular subject matter is." Of the several item that are common to all being, some have a special status, however, they are those features that can be separated from matter and change, both in thought (or "account") and reality.

Aristotle builds up to this claim by means of two examples. His first example concerns the distinction of metaphysics and the natural sciences. He writes that:

> [The natural sciences] deal with a special kind of being, namely, the sort of primary being which has its source of movement or rest in itself. (1025b18–20)

This entails that, essentially, the objects of natural sciences cannot be considered without reference to the material in which their form is embodied and the change it undergoes. Using his famous 'snub' example and the notion that matter itself is responsible for change, Aristotle notes:

> If all natural entities, therefore, are like the "snub", we cannot define or explain [natural entities] without reference to movement, since they are always material. (1026a1–3)[11]

In general, Aristotle believes that the subject-genus of the natural sciences is the type of entity that is inseparable from its material embodiment, either in thought—how we "define or explain it"—or reality—in the mode of its actual existence. It is a genuine hylomorphic unity.

Aristotle's second example concerns mathematics. Whereas the natural sciences consider entities that have within them their source or movement of rest (i.e., entities that necessarily change), mathematics considers entities that are "immovable", entities that do not change. Although at this point in *Metaphysics E* Aristotle's conception of mathematics has not been fully elaborated, by *Metaphysics M* we find out that mathematics is the study of those

[11] For a general examination of this central Aristotelian example, see Balme 1984.

features of objects that can be considered independently of their material embodiment and change, although in actual existence they are not independent of their material embodiment and change.[12] Mathematics, in a phrase, considers what is separable in thought, but not in reality. Aristotle merely alludes to this in book *E* (1026a9), but uses the idea to articulate the difference between metaphysics, mathematics, and natural science.[13]

Metaphysics is distinct from both the physical sciences and mathematics, Aristotle writes, because it considers those features of objects that are separable in *both* thought and reality. In other words, metaphysics is the study of those features of objects that can be considered independently of their material embodiment and change *and* that are actually independent of their material embodiment and change. The objects of metaphysics, in other words, are those that are both "immovable and independent" (1026a13).

It should be clear now that metaphysics, physics, and mathematics constitute different genera because they deal with essentially different objects. Whereas metaphysics deals with objects that are "both independent and immovable" and the natural sciences deals with objects that are "neither independent nor immovable," mathematics deals with objects that are "immovable, [but] are for the most part not independent of material reference" (1026a14–16).[14] It is for this reason that Aristotle believed that mathematics is inherently inadequate for explaining natural phenomena: mathematics ignored what is most important about physical objects: that they are the kind of objects that change, and moreover, that they are the kind of objects that have within them their source of change.

[12]See, e.g., Annas 1976: 26–41, Hussey 1991, Modrak 1989.

[13]I am ignoring here an important aspect of Aristotle's characterization of physics, mathematics, and metaphysics. In *Metaphysics E*, Aristotle not only distinguished between the three, but argued that each is *theoretical* (as opposed to practical or productive). The idea that physics, in particular, is a 'theoretical' or 'speculative' discipline lasted well into the early-modern period and was one of the points which against some *novatores* of that sixteenth and seventeenth centuries rebelled. Particularly within the humanist tradition, it became increasingly common to demand that human knowledge produce 'works'; that is, that it deliver goods that can be practically used in the life of the individual and/or the society. Starkly (and over-simply) put, whereas the central goal of sciences in the scholastic-Aristotelian tradition was to provide an intellectual understanding—a grasp, if you will—of their subject matter, their goal in the humanist tradition was to produce commodities.

[14]A different reading of the distinction of physics and mathematics is given by Annas 1976: 29ff.. I'll get to it shortly.

2.1.3 GENERA CROSSING: THE MIXED-MATHEMATICIAN'S MODE OF THOUGHT

How, then, can mathematical and physical objects be *somehow* the same? As essays by McKirahan 1978 and Lennox 1986 make clear (although they differ in detail),[15] the key to understanding this "somehow" lies in the mixed-mathematician's mode of thought, a mode that can bridge the generic gulf created by the ontological structure presented in *Metaphysics E*.

According to McKirahan 1978, the mixed-mathematician must use a *modus considerandi*—to follow Zabarella's phrase—that simultaneously keeps before his mind both separable-in-thought mathematical features and the matter in which they are embodied, the matter in virtue of which those mathematical features are true of the object in question, but the matter that is accidental to the mathematical features *per se*. In this way, s/he can construct syllogisms that supply a mathematical account of some natural phenomena, despite the fact that mathematics, taken *per se*, does not latch onto what is essential about the natural world.

Lennox 1986 offers a different analysis of this *modus considerandi*. According to Lennox, the distinguishing characteristic of a subordinate science is that it involves "applying to physical objects our recognition that various physical objects have certain mathematical features in virtue of their (basic) mathematical structure (p. 41)." In a similar fashion, Lennox characterizes the type of syllogism appropriate to a subordinate science as one whose "middle term picks out the description of a natural object in virtue of which it has a certain mathematical property; that property is a per se property of a natural kind qua being a mathematical kind (p. 41)." What becomes clear in Lennox's overall presentation is that although a subordinate science reasons mathematically, it does so *only* about objects whose physical nature is such that particular mathematical descriptions can be true of them. The *modus considerandi* is to look at natural objects as if they were mathematical, as if they lacked all those features that make them natural, but only having already recognized that they belong to a natural kind that makes mathematical description of them appropriate.

A crucial difference between McKirahan and Lennox is that Lennox describes the business

[15]Lennox is further buttressed by Lear 1982.

of the mixed-mathematician as studying *proper attributes* (specifically, mathematical ones) of natural objects, not their form (with "form" understood mathematically). Although it seems to me that Lennox insists on this point for precisely those reasons concerning disciplinary classification which will be rehearsed in the second portion of this chapter, taking a stand on this difference is not crucial for my argument in this chapter. Whether the business of the mixed-mathematician is to study proper mathematical attributes or form mathematically understood, the central insight shortly described—that the mixed-mathematician cannot do either in isolation from material considerations—will still stand.

Notice, however, that although the description of the mixed-mathematician's way of thinking differs in McKirahan and Lennox, they both fix on a central point: that a mixed-mathematician cannot entertain mathematical features of natural objects (as elements of form or proper attributes) in utter ignorance of their non-mathematical, natural features. In both cases, the authors justify this central point (and further address questions such how can physical objects be considered *qua* mathematical and how can they be looked upon as mathematical *and* embodied at the same time) by appealing to Aristotle's theory of abstraction and separability. Aristotelian abstraction is a process of stripping away features of objects—deliberately failing to pay attentions to those features—until one is able to posit an object that possess only a certain set of features.[16] To arrive at a mathematical object, one must begin with an actual object and strip away from it all properties that are not mathematical. Once this is done, one is left is an object that is purely mathematical, although that object does not have actual independent existence. The mathematical object then comes to stand for the actual object. As long as one sticks to mathematical reasoning, any inferences drawn regarding the mathematical object applies equally to the actual object. Of course, this is but a sketch and the nature of abstraction is more complicated that it suggests. For example, debates exist concerning the results of abstraction (whether they are objects or bundles of properties) and their ontic status (say, if they are objects, whether they are fictional or not).[17]

[16]This reading of abstraction follows Lear 1982, although Lennox 1986 objects to some central tenets, such as the *fictional* character of the results of abstraction, of the former.

[17]See, e.g., Lear 1982 and Lennox 1986 on the "fictional" status of abstracta and Annas 1976 and Lennox 1986 on their metaphysical makeup. See also Baxter 1997, Mueller 1990.

Two matters are important here. The first concerns a side-project that has been weaved into this chapter thus far; namely, that of articulating different criteria by which Aristotelians may distinguish branches of knowledge. Aristotle's theory of abstraction suggests to some a third criterion for doing this. The most forceful advocate of this criterion, as far as I am aware, is Julia Annas. She holds that because mathematical entities are arrived at by a process of abstraction, the difference between physics and mathematics boils down to a difference in the cognitive process by which human beings arrive at those sciences, *not* a difference in the objects with which these sciences are concerned.[18] She argues that mathematics is distinguished from natural science "not by its subject-matter but by its method" (Annas 1976: 29). To the extent that their objects are different, she holds, the difference is traceable to the cognitive process by which these objects are generated or recognized. In the language I have used above, it seems that Annas endorses (in regards to Aristotle) a *Methods Criterion*, the idea that areas of knowledge are individuated according to the procedures human beings follow in order to pursue them.[19] These criteria will play a role in the next chapter, when they are made use by the late scholastic to distinguish sciences.

The second matter of importance is that Aristotle's theory of abstraction—regardless of the way its details are fleshed out—is neutral with respect to the central insight of Lennox and McKirahan concerning the *modus considerandi* of the mixed-mathematician. Regardless of what mathematical abstracta consist in, that insight is that the mixed-mathematician must somehow bridge those abstracta and the other features of the entities from which they are abstracted. To put it another way, it is the idea that a mixed-mathematician cannot entertain mathematical features of natural objects in utter ignorance of their non-mathematical, natural features. It is this feature of the mixed-mathematician's *modus considerandi* that I'd like to call the "tempered-hylomorphism" of the mixed-mathematical sciences.

I call this "*tempered*-hylomorphism" because the objects with which the mixed-mathematician must be concerned are not fully irreducible composites of form and matter, as a *non*-qualified hylomorphism might suggest. Rather, they are *tempered* composites—composites whose components can be understood individually, but not fully so. In order to better articulate

[18] Annas 1976: 29ff..

[19] Annas' account is problematic. Particularly, the claim that abstraction does not play a similar role in natural science and mathematics is contentious and runs contrary to more recent scholarship.

and defend this conception, I'll take Aristotle's statements regarding the mixed-mathematical sciences in *Physics* II.2 as a case in point. I'll argue that they support the central insight discussed above, allow us to flesh it out further, and, most importantly, explain the overall intent of *Physics* II.2 in a way that is difficult to see without keeping in mind the tempered-hylomorphic character of the mixed-mathematical sciences. Questioning this overall intent will be precisely how the tempered-hylomorphism of the mixed-mathematical sciences will come into view.

2.2 TEMPERED-HYLOMORPHISM: A CASE STUDY OF *PHYSICS* II.2

After setting out the character and structure of the *archai* appropriate for natural inquiry in *Physics* I, Aristotle makes a fresh start in *Physics* II.1 on a fundamental question of this inquiry—namely, what is nature?[20] After offering a preliminary analysis of the problem, he begins a seemingly separate chain of reasoning with the following words:

> Having distinguished the various things which are called nature, we must go on to consider how the student of mathematics differs from the student of nature (*Physics* II.2 194b22).

The subsequent short discussion is difficult to interpret. First, Aristotle gives no clear indication whether the conclusions reached in it play an important role for the fundamental question of *Physics* II.1. Second, its structure is surprising: it seems to be interrupted by a repeat discussion of the topics of *Physics* II.1 before continuing on its seemingly separate trajectory. Third, its argument can be read both as offering a dichotomy between the study of mathematics and the study of nature *and* suggesting that their difference might be more nuanced, with some branches of mathematics being more like the study of nature than others.

The goal of the present section is to work up to an understanding of the character of the mixed-mathematical sciences and thus the distinction between mathematics and natural science in *Physics* II.2 in light of the fact that some sciences—the mixed-mathematical ones,

[20] I am much indebted to Jim Lennox for discussion regarding this interpretation of *Physics* II.2 and for sharing with me drafts of his complementary work on the same subject. His paper is forthcoming in *Oxford Studies in Ancient Philosophy* (2008), as "'As if we were investigating snubness': Aristotle on the prospects for a single science of nature".

the so-called "more natural branches of mathematics"—seem to straddle mathematics and natural science. I will pursue this goal by attempting to clarify the importance of these topics to the larger questions of *Physics* II concerning nature, matter, and form. To preview, I will argue that Aristotle's appeal to the "more natural" of the mathematical sciences ought to be understood as a careful recommendation to the reader: in order to understand the mode of investigation of natural science as a whole, one must consider the mode of investigation of the "more natural" branches of mathematics. Aristotle does not suggest that these branches reveal *all there is to know* about the character of natural science, but rather that they reveal a single, yet crucially important feature of it: that it is *essential* for natural science to study *both* matter and form. The argument of *Physics* II.2 also shows that there are ways in which the "more natural" sciences are an *inappropriate* model for natural science: although they suggest that the conjunction of matter and form is essential, they fail to adequately represent their *union in natural substance* or the needs for natural science to investigate matter's well-suited-ness for the expression of form. In a sense, in *Physics* II.2 Aristotle offers the reader the easier to digest tempered-hylomorphism of mixed-mathematics in order to clear the way for the full hylomorphism of the natural sciences.

2.2.1 THE ARGUMENTATIVE STRUCTURE OF *PHYSICS* II.1–2

Aristotle begins his inquiry into what nature is in *Physics* II.1 by introducing a supposedly well-known distinction between things that are due to nature and things that are due to other causes. By considering both sorts of items, Aristotle uncovers the implicit standard according to which the distinction is drawn; namely, that things that are due to nature have within them "a source of change and staying unchanged". This source belongs to them primarily, not accidentally, by virtue of the sorts of things they are (192b9–27). Having established this, Aristotle continues to consider the ways in which "nature" can be understood; that is, the ways in which the "source of change and staying unchanged" can be attributed to a thing given that thing's account. He offers two options: the source of change can be identified with a thing's "primary underlying matter" (193a9–30), and/or it can be identified with a thing's "shape and form" (193a28–193b18). The chapter concludes with what is clearly the most controversial question: whether "the primary underlying matter" or "the shape and form" has the better claim to being a thing's nature (193a9–193b22). Aristotle comes down decisively on the side of the latter (193b7–9), and adduces arguments that offer additional ways in which a thing's "nature" can be spoken of as its form (193b10–18).

Physics II.2 continues the arc of II.1 by defining the nature and scope of an *inquiry* into nature. The chapter begins by distinguishing natural science from mathematics (193b22–194a12) and closes by distinguishing natural science from theology (194b15). This way of arguing not only accounts for all of Aristotle's theoretical sciences, it does so by registering natural science's *differentiae* with other field of studies, thus following the procedure for arriving at a definition recommended in the *Posterior Analytics* I.2 & 3.[21] What's surprising, however, is that between the treatments of mathematics and theology the chapter seems to return to the theme of *Physics* II.1—the interrelation matter and form—*without drawing any explicit morals from the interposing discussion regarding the character of natural science* (194a13–194b15). In fact, Aristotle's thesis in this part of the chapter—that the student of nature ought to study both matter and form, but primarily form—mirrors almost exactly the thesis of chapter one—that *nature* must be conceived of as both matter and form, but primarily form.

This structure is problematic. Even if one could ignore *Physics* II.2's unnecessarily staccato arguments about disciplinary boundaries (193b22-194a12, 194b15), it is still unclear why in the second main section of II.2 Aristotle would need to draw the seemingly obvious inference that the study of nature has as its subject-matter *nature*, precisely as defined in

[21] In the commentary tradition, the limning of disciplinary boundaries is most often cited as the purpose of *Physics* II.2. Philoponus, for example, writes:

> [Aristotle] wants next to distinguish (what pertains to) the study of nature from (what pertains to) mathematics and (what pertains to) theology; for it belongs to the man with special knowledge [i.e., a practitioner of this or that discipline] to set apart, when delineating the matters which are relevant to him, those which seem to be relevant but are not really so (Philoponus 1993: 218,25–219,4; parentheses added by translator, square brackets are mine).

Both Ross (1966: 506) and Charlton (1970: 93) are remarkably terse regarding the rationale behind the initial *aporia* of *Physics* II.2, choosing only to state it in terms of disciplinary boundaries without explaining its presence. Notably, Simplicius does not focus on disciplinary distinctions when interpreting *Physics* II.2. I will return to his interpretation—which I believe is correct—shortly.

Physics II.1 (194a13–194b15). One may argue that the purpose of the discussion is merely to state, not to argue for, the subject matter of natural science. However, if this is the case, why is the discussion not part of Aristotle's arguments in *Physics* I regarding the very existence of a distinct enterprise concerned with natural change? This incongruity suggests that although it is clear that *Physics* II.2's purpose is to delineates the subject matter of natural science, more must be made of its relationship to II.1 in a way that sheds light on the need to have a chapter devoted to this purpose at this point in the Physics.

To shed light on this problematic structure, I believe we must consider two intertwined but distinct features of *Physics* II.1: Aristotle's arguments for the priority of form over matter, and the notion that forms are inseparable from matter. Near the end of *Physics* II.1, Aristotle offers three arguments for the thesis that form has the better claim to be called a thing's "nature" than its matter—from language (193a32–193b3), from generation (193b8–12), and from growth (193b13–18). At the same time, however, he mentions but does not elaborate on the claim that form is "not separable except in respect of its account" (193b6). Although it is clear that the clause provides another way of articulating an idea already discussed in *Physics* II.1—that the source of a thing's change must belong to it "primarily and of itself, that is, not by virtue of concurrence" (192b23)—Aristotle has yet to relate the notion of *separation* to questions of matter and form. In fact, his three arguments for the priority of form over matter can be easily interpreted to suggest that form is entirely independent from matter! Because of this, it seems that an unstated difficulty arises right at the close of *Physics* II.1; namely, how does one reconcile form's inseparability from matter with its priority over it? If form and matter are inextricably joined, how can one be said to dominate the other? As I will show in the next section, this would have been an obvious question for Aristotle's audience, who would have been tempted to locate a thing's "nature" in *either* its form *or* its matter, but would have rejected the type of reconciliation Aristotle was seeking.

My claim is that the implicit purpose of *Physics* II.2 is to provide a more nuanced understanding of form's relation to matter and to ward off attempts to reject a reconciliation. In particular, *Physics* II.2 supplies the missing discussion of separation precisely at that point in the *Physics* where Aristotle's contemporaries would have likely misinterpreted the idea that form is "inseparable except in account". Moreover, it provides a host of exemplars by which to come to terms with form's seemingly janus features. In this sense, the chapter is not so much concerned with delineating the *subject matter* of the study of nature as it is with providing additional evidence from other fields of study (the mathematically subordinate sciences, and the arts) which sheds light on the *modus considerandi* appropriate for natural scientist—a mode that must keep a firm grip both on form's inseparability from matter and priority over it. The focus on the interrelation of matter to form in *Physics* II.1 and II.2 thus forms a continuous whole. The interpretive problem is not to explain why the latter half of *Physics* II.2 echoes *Physics* II.1 in a seemingly staccato fashion, but to understand how II.2's apparently disparate sections compose two themes of a single legato movement.

2.2.2 THE PLATONIST AND THE CONTRAPUNTAL STRUCTURE OF *PHYSICS* II.2

The suggestion that *Physics* II.2 ought to be understood as providing exemplars for the mode of investigation appropriate for natural science entails that we should not expect to find the chapter arguing exclusively for theses which would have been novel to Aristotle's contemporaries. Rather, we should find in it an appeal to an already understood context, perhaps with a new twist. In fact, chapter two's opening salvo does just this. In it, Aristotle offers a short *aporia* which presupposes that "the business of the mathematician" is well known and likely to be different than the business of the natural scientist:

> [W]e must... consider how the student of mathematics differs from the student of nature—for natural bodies have planes, solids, lengths, and points, which are the business of the mathematicians. And again, is astronomy a branch of the study of nature of a separate subject? It would be absurd if the student of nature were expected to know what the sun or moon is, but not to know any of the things which of themselves they have supervening on them, especially as it is plain that those who discuss nature do also discuss the shape of the sun and the moon, and whether the earth and the cosmos are spherical or not (193b22–31).

How would Aristotle's audience have understood "the business of the mathematician"? Certainly, they would have assumed the conception of mathematics endorsed by the Academic Platonists. I will focus on Plato directly.

Put shortly, the business of the Platonic mathematician is to 'discover' (or perhaps, more accurately, 'recollect') features of objects that, in themselves, are genuinely existing non-perceptual entities (e.g., *Meno* 81a-86a, *Pheado* 73a-77a, *Euthydemus* 290b–c, *Pheado* 103–106, *Republic* 527a-d). Whether or not Plato conceived of mathematical objects as full-fledged Forms, it is clear that for him mathematical objects are matter-less and possessed of only formal characteristics.[22] To the extent that material bodies exemplify mathematical objects, it is because these mathematical objects are imperfectly instantiated in those bodies (*Republic* 527a-d). Mathematical objects thus have priority over matter precisely because without them, material bodies could not exemplify mathematical structure at all. The creation story of the *Timaeus* further imbues this conception with force and color by asserting that to the extent that the material universe has any structure at all, it is because it had been purposely modeled after pre-existing mathematical forms.

However, if one has this conception in mind, one can easily misinterpret the conclusions of *Physics* II.1 in a way that suggests that the student of mathematics is *no different* than the student of nature. The Platonist, focusing on chapter one's insistence on the priority of form over matter and its possible separation in respect of account, may understand Aristotle's forms to be Platonic in character and thus believe the mathematician and the natural scientist study the same things: forms separated from matter. As Charlton (1970: p. 96) notes in his commentary on the the *Physics*, "[t]he tendency of the Academy was to confine the study of nature to the study of the *forms* of natural things," precisely what Aristotle is ultimately arguing against (emphasis added). Aristotle recognizes the possibility of such an interpretation in his very characterization of mathematical practice:

> Both the student of nature and the mathematician deal with these things [i.e., planes, solids, lengths and points]; but the mathematician does not consider them as boundaries of natural bodies. Nor does he consider things which supervene as supervening on such bodies. That is why he separates them; for they are separable in thought from change, and it makes no difference; no error results (193b32–194a2).[23]

[22] cf. Annas (1976: p. 13), who throws doubt on the possibility of understanding Plato's conception of numbers as identical to his conception of other Forms (say, the beautiful and the white).

[23] Hardie and Gaye (in Aristotle 1971: 331) translate the passage as follows:

> Now the mathematician, though he too treats of these things, nevertheless does not treat them as the limits of a natural body; nor does he consider the attributes indicates as the attributes of such bodies. That is why he separates them; for in though they are separable

However, Aristotle holds the same mode of investigation *cannot* be appropriate for natural science. This is not because he believes this mode of investigations is flawed *simpliciter*—in fact, both the Platonist and Aristotle can find some merit in it.[24] Rather, it is because he believes the mathematician's practice cannot do justice to natural things.

To make this point, Aristotle returns to the notion of separation. The topic was not addressed in the closing passages of *Physics* II.1 because the dominant theme there—that form has priority over matter—would have already been attractive to the Platonist. In chapter two, however, Aristotle is trying to reign in the Platonic intuitions of his readers, and thus it is precisely at this point that it become crucial to differentiate his 'separation' from that of Platonists. The following oft-discussed passage thus serves as a prophylactic against a reduction of form's complex structure to one of its seemingly opposing characteristics:

> Those who talk about ideas do not notice that they too are doing this: they separate physical things though they are less separable than the objects of mathematics. That becomes clear if you try to define the objects and the things which supervene in each class. Odd and even, straight and curved, number, line, and shape can be defined without change but flesh, bone, and man cannot. They are like snub nose, not like curved (194a2–7).

> from motion, and it makes no difference, nor does any falsity result, if they are separated.

This translation is preferable to Charlton's insofar as it does does not invoke the neologism "supervene" to characterize the relationship of the attributes of bodies to the bodies of which they are attributes. However, I chose to cite Charlton in the text because his "things which supervene" remain neutral with respect to the character of objects which the mathematician studies, while Hardy and Gaye insist that they are "attributes". Although I agree with Hardy and Gaye that this is ultimately the business of the mathematician, I follow the neutral formulation in order to straddle the alternate formulations of the business of the mixed-mathematician offered by Lennox and McKirahan in §2.1.3.

[24] Of course, the Platonist would not agree with Aristotle's claim that mathematical objects (or properties) are "things which supervene" on physical objects. Regardless, both would consent to the final sentence: "That is why he separates them; for they are separable in thought from change, and it makes no difference; no error results". It just so happen that the Platonist would consent to it because he believes the objects of mathematics are separable in thought *because* they are separate in reality.

But we must not make too much of this difference in the present context. Aristotle is concerned only with separation in thought here—that is, with the *practice* of the mathematician, not with the underlying metaphysics that justify the practice. In other words, I am suggesting that one ought not to interpret this and the subsequent passages of *Physics* II.2 as an abbreviated version of the argument of *Metaphysics M*. Although the conception of mathematics of the *Metaphysics* is surely endorsed here, the point, as I will continue to argue, is not regarding the ontological commitments of the pure mathematician, but regarding the conceptual structure of the natural sciences (which is partly explicated with the help of the subordinate sciences). For this reason, attempts to find in this passage a realist, non-Platonic conception of pure mathematics are successful, but misguided. The passage supports such interpretations, but Aristotle does not want to combat the ontological underpinning of the Platonic mathematician in the present context—he *does* allow the mathematician to separate his forms *entirely*, and the question of whether this is a justifiable separation only in thought or both in thought and reality is tangential. The focus on *thought* continues throughout the remainder of the passage: the snub cannot be *defined* without change, geometry *considers* natural lines not as natural. We will return to this topic in §2.2.3.

The closing dilemma of *Physics* II.1—between priority and inseparability—is also vulnerable to yet another equally extreme misinterpretation, *particularly in light of the above passages*. Aristotle's claim that a natural scientist cannot study form in complete separation from matter and change (because errors *do* result from such a separation) can suggest that insofar as a natural scientist is a student of *nature*, it is because s/he studies *enmattered* form. In other words, one may be pushed back into the Empedoclean position of locating a thing's nature—its "source of change and staying unchanged"—solely in the material portion of the form/matter composite. On this interpretation, although the natural scientists studies both matter and form, insofar as s/he studies form, s/he does so in the same fashion as the mathematician—as completely separated from matter and change. It would only be to the extent that separated forms are reintroduced into matter that their study takes on the unique character of the study of *nature*. Consequently, although *Physics* II.1 has argued forcefully against the Empedoclean position, Aristotle's anti-Platonic discussion leaves room for its re-introduction.

It is for this reason that the closing theme of *Physics* II.1 one must be recapitulated in the latter half of II.2. Aristotle must once again stress that the "source of change and staying unchanged" can be found in form *qua* form, not merely form *qua* enmattered form. Of course, Aristotle would not claim that a form could be a source of change entirely separately from matter, but neither would he claim that it is a source of change only insofar as it is enmattered. The point is that form can be a source of change *qua* being a form. Consequently, the student of nature must study it as a source of change in itself. The closing discussion of *Physics* II.2 goes to stress just this by noting the ways in which the study of matter, insofar as it is relevant to the study of change—i.e., "up to a point" (194a22)—is not in itself a study of change, but is incumbent on the study of form *as the source of change*. Several arguments are offered stressing either that change is driven by form (e.g., 194a27–33) or that matter can only be understood in relation to form (194a33–b9). In this section, the dominant examples come from the field of art.

The structure of *Physics* II.2 as a whole is therefore contrapuntal, with an original theme sounded by the discussion of mathematics and a counter-theme posed by lessons drawn from art. Both themes are attempts to elaborate the leitmotif of *Physics* II.1: that nature must be conceived of as both matter and form, but primarily form. At the same time, they go to constrain extreme positions which hold on to either form's priority over matter or form's inseparability from matter, but not both. To use a different metaphor, we might say that the overall argument of *Physics* II.2 acts as a pincer, seeking to center the proper interpretation of form between the two extremes just mentioned. The fulcrum of this pincer, as the next section argues, is Aristotle's discussion of the subordinate, mixed-mathematical sciences.

2.2.3 THE SUBORDINATE SCIENCES: A SEPARATE QUESTION?

Thus far, we've only considered Aristotle's discussion of what we would call the 'pure' mathematician—the one for whom forms "are separable in thought from change, and it makes no difference". However, Aristotle also provides examples from other branches of mathematics which flesh out his critique of those who would identify the mode of investigation of the pure mathematician with the mode of investigation of the natural scientist:

> The point is clear also from those branches of mathematics which come nearest to the study of nature, like optics, harmonics, and astronomy. They are in a way the reverse of geometry. Geometry considers natural lines, but not as natural; optics treats of mathematical lines, but considers them not as mathematical but as natural (194a75–12, emphasis added).

The phrase "[t]he point is clear also" indicates that Aristotle takes optics, harmonics, and astronomy to supply additional evidence for the anti-Platonic point he had just articulated. The mode of investigation of optics, harmonics, and astronomy must therefore be understood as importantly *un*like the mode of investigation of pure mathematics *in that the former is not prey to the critique leveled against the latter.* The objects of the subordinate sciences must be in a significant way like flesh, bone, man, and the snub nose: it must be impossible to define them without reference to matter. Aristotle's overall discussion of mathematics thus contains an inversion between a critique of certain branches of the field and an endorsement of others (insofar as they are instructive for understanding the relationship between matter and form in natural science). The inversion is also emphasized by the statement that the subordinate science are "the reverse" of pure mathematics; i.e., they are opposite in respect of the discussion just concluded.

A direct implication of this reading is that although the two questions in the chapter's initial *aporia*—"how [does] the student of mathematics differs from the student of nature[?]" and "is astronomy a branch of the study of nature[?]"—seem like increasingly accurate specifications of the *same* question, they receive significantly different treatments and should be understood as distinct. This is by no means an unobjectionable claim. Ross (1966: p. 506), for example, claims that "both questions are treated together, i.e., the general question [regarding mathematics] is treated with special reference to astronomy". Charlton (1970) and Philoponus (1993) do not even discuss a possible difference between the questions.

In general, it seems the most direct *internal* evidence for this interpretation is the structure of *aporia* itself. In setting out the question(s?), Aristotle notes that the student of nature examines "the shape of the sun and the moon, and whether the earth and the cosmos are spherical or not (193b30-31)"—topics which seem to be in the purview of the *astronomer* (*De Caelo* II. 14). Immediately thereafter, he elucidates the statement with "both the student of nature and the *mathematician* deal with these things" (193b32-33, emphasis added)—thereby seeming to equate the question concerning the mathematician and the one concerning astronomy. However, if the previous section is correct about the overall purpose of *Physics* II.2, then at the outset of the chapter Aristotle is leading his Platonic readers by the nose so that he can expose and discipline their intuitions. Consequently, it seem plausible from a rhetorical point of view that the two questions are framed as increasingly accurate specifications of the same question because a Platonist—one for whom mathematics, natural science, and the subordinate sciences embody the same mode of investigation—would expect their *answers* to be the same. However, as I will be argue shortly, Aristotle's treatment of astronomy differs from his treatment of pure mathematics precisely in a way that exposes the initial error of the Platonist. We must therefore turn to these answers.[25]

B

In relation to the first question, it is clear that Aristotle believes the student of pure mathematics differs from the student of nature because the former can justifiably separate

[25]As would be expected, the commentators noted above all believe that Aristotle provides the *same* answer to the two questions—namely, that neither mathematics nor astronomy are branches of natural science—and thus reasonably hold that only one question is at stake. Of course, those who believe that Aristotle assimilates astronomy but not mathematics to natural science reasonably believe that *two* question are at stake (e.g., (Aquinas 1963: Lecture 3)). In the following paragraph, I argue for a third option.

in thought the objects of his/her study from matter, while the latter cannot. In relation to the second question, the situation ought to be "the reverse". The student of astronomy ought *not* to be able to separate the the objects of study from matter *in thought*—they ought to be like the snub. Does this make astronomy a branch of natural science? Not quite. Aristotle is clear in a variety of other contexts that astronomy is a branch of mathematics (e.g., *Metaphysics M* 1078a14–16, *E* 1026a12-14, *K* 1059b10–12, etc.) and thus (given the theory of demonstration of the *Posterior Analytics*) cannot also be a branch of natural science. Even in the present context, he does not assimilate astronomy to the study of nature, but explicitly refers to it as a branch of mathematics. Instead, we should understand that although neither astronomy nor mathematics are branches of natural sciences, there is *something in astronomy whose understanding is relevant to the study of nature.* Aristotle's answer to the second question can therefore be read as a qualified 'no', as a '*somehow* no'. Although astronomical investigation "treat[s] of mathematical lines" and is thus a solidly mathematical activity, it does so in a way that resembles the treatment appropriately given to natural bodies. What could this be? Given what we have said about separability, we should expect to find some sense in which the objects studied by the subordinate, mixed-mathematical sciences, like the objects studied by natural science and unlike mathematical objects, are inseparable *even in thought.*

2.2.4 The Disciplinary Integrity and Nature of the Subordinate Sciences

We can now return to the characterization of the mixed-mathematical *modus considerandi* begun in §2.1.3. As noted there, what I called the central insight of McKirahan and Lennox is that a mixed-mathematician cannot entertain mathematical features of natural objects (formal elements or proper attributes) in utter ignorance of their non-mathematical, natural features. The point stressed there was that it is the non-mathematical, natural features of objects that allow for recognition that, to use Lennox's language, a certain natural kind is also a mathematical kind. However, we are now in a position to reveal another crucial motivation for the central insight. It is that if a physical nature allowing for the application of a mathematical description is forgotten, a subordinate science collapses into the mathematical science to which it is subordinated. Take optics for example: geometrical reasoning applies to visual rays because when considered as a mathematical kind, the natural kind "visual ray" has a certain mathematical structure. Such reasoning constitutes *optical* reasoning. If, however, it is forgotten that optical reasoning is applicable *because* the kind "visual ray" has a certain mathematical structure (or more simply, because light rays have a certain mathematical structure), then optical reasoning becomes nothing more than geometrical reasoning. In this sense, one cannot do *optics* if one does not keep the natural kind "visual ray" in mind. Similar argument apply to astronomy and harmonics.

What is special to the subordinate sciences is therefore that their disciplinary integrity disappears if the forms they study are considered separately from the natural objects of which they are proper attributes. In other words, the subordinate sciences lose their character *as subordinate sciences* if their objects of study are considered apart from matter. To be clear, the claim is not that the objects of the subordinate sciences must be *ontologically* inseparable—for Aristotle, this is true even of purely mathematical objects. Rather, the claim is that these objects cannot be separated *even in thought or account* without failing to be the type of objects they are. So, to repeat, even though the forms or proper attributes of the subordinate sciences are mathematical forms or proper attributes—and thus as separable in thought as the objects of pure mathematics—the difference between the mathematicians and the subordinate scientists lies in the fact that the subordinate scientists must always keep both mathematical form or proper attributes and matter firmly in mind, *despite the fact that mathematical form or mathematical proper attributes are themselves separable.*[26] This is precisely how Philoponus understood this portion of the *Physics*. He wrote:

> [G]eometry and arithmetic simply look at double and one-and-a-half and such [relations] separably, while the harmonicist, since he studies such formulas (*logos*) as actually being properties [of something], cannot even think of them without matter (Philoponus 1993: 227,1 9–13).

[26]A strain of scholarship concerning Aristotle's philosophy of mathematics (best exemplified by Mueller (1970)) argues for the superficially similar thesis that for Aristotle all purely mathematical objects are enmattered. However, Mueller focuses on the notion of *intelligible* matter—the incorporeal matter of purely mathematical objects—while the above discussion concerns *perceptible* matter (see *Metaphysics* Z.1 1036a9–12). My claim is that when considering the objects of the subordinate sciences one must conceive of them in perceptible matter, not simply intelligible matter. Consequently, while it may be possible to unify Aristotle's treatment of pure mathematics and the subordinate sciences via the notion of 'intelligible matter', such a unification would not shed light on this portion *Physics* II.2. This reading is more consonant with the interpretation of Aristotle's philosophy of mathematics offered by Lear (1982).

And this is precisely what I mean by the tempered-hylomorphism of the mixed-mathematical sciences. It is the claim that the mixed-mathematician must study mathematical features as mathematical features of some natural, enmattered, kind, on pain of losing his/her disciplinary integrity as a mixed-mathematician, although the mathematical features, taken in themselves, are separable. And finally, because the modes of investigation of mathematics and the subordinate sciences are different in this way, it would be heavy handed to read the questions of *Physics* II.2 as treating them uniformly.[27]

2.2.5 A QUALIFIED RECOMMENDATION

Given the intimate connection between matter and mathematical features in the subordinate sciences, we see why Aristotle appeals to the subordinate sciences in order to illuminate the mode of investigation appropriate for natural science. Like the subordinate scientist, the natural scientist must always keep firmly in mind both matter and form, on pain of loosing sight of the very nature of his/her investigation. *This* is a helpful way of conceiving the connection between the *archai* of natural science, as described in *Physics* II.1. Moreover, the priority of the mathematical form in the demonstrations of the subordinate sciences suggests that although matter and form cannot be separated, it is still on form that the student of nature must focus his/her attention, albeit not exclusively. For these reasons, its seems Aristotle's reference to the "more natural" of the branches of mathematics should be taken as a recommendation to his audience of Academic Platonists: keep in mind the subordinate sciences in order to understand something of the character of the natural sciences. To quote another ancient authority, this is the reading given to this passage by Simplicius:

> It becomes apparent that scientists engaged in optics, harmonics, and astronomy cannot separate their subject matter from natural bodies when each of them is actively engaged in research and *applies himself to his proper business*... If, then, it is not always possible to separate lines, numbers and circles although they are in definition separable, *how much truer is this of natural entities and compounds themselves*, whose definition are given as those of natural entities and compounds? (294,34–295,11, emphasis added)

Once again, tempered-hylomorphism comes to the fore. To put it in yet another way, it is the claim that the mixed-mathematician cannot separate his mathematical subject matter

[27] See also footnote 24 in light of this discussion.

from natural bodies when he “applies himself to his proper business”.

Yet the subordinate sciences cannot give Aristotle all he wants. In particular, they do not make clear that matter itself must be investigated to a sufficiently high degree in natural science, and investigated *in relation to* form. In fact, apart from appealing to a cognitive attitude that keeps both form and its material embodiment in mind, the subordinate sciences do not investigate at all the matter in which mathematical features inhere—it is for this reason that they appear so close to pure mathematics. In order to clarify that matter must not only be kept in mind, but investigated *as intimately connected to a source of change*, Aristotle must furnish additional examples and argumentation. These constitute the remainder of *Physics* II.2. The exemplar of the subordinate sciences is thus the turning point of the chapter. After it, Aristotle turns to consider the way in which *matter* ought to be studied in the context of natural science. He does so precisely because the subordinate sciences do not illuminate the deeper natural dependencies of matter and form. Put differently, while the subordinate sciences show that both matter and form must be studied by natural scientists, they do not show that this study must be a single, unified activity. The two portions of *Physics* II.2 can thus be seen as answering two concerns. The first demonstrates that both matter and form are essential to the study of nature; the second demonstrates that the study of matter and the study of form must not only be individually necessary and jointly sufficient, but that a *unified* study of both is necessary and sufficient. It is for this reason that he asks whether “both [matter and form] fall under the same study, or each under a different (194b17)”.

In light of this, the interpretation previously given to the latter half of chapter two can be supplemented. Not only must Aristotle try there to ward off a return to an Empedoclean position, he must temper his response by emphasizing the significant role that ought to be given to the study of matter in connection with the study of form within natural science. In other words, he must stress that the study of matter cannot be all that there is to natural science, but neither can it be just a subsection of a larger enterprise: it must be intertwined with the study of form. Consequently, his arguments in this section stress *both* that the study of matter is dependent on the study of form *and* that it is still important to investigate matter *somewhat* independently (194a21–27, 194a33–b7, 194b10–14). To what extent ought one investigate matter independently? "[U]p to a point". The question of how to understand "up to a point" is beyond the scope of this chapter, but it should be clear why Aristotle resorts to the phrase. In trying to shed light on the relation of matter and form described in *Physics* II.1, Aristotle uses the subordinate sciences as an example of an investigation in which form cannot be separated from matter. However, this example requires that he devote additional time to showing how natural science is *different* from a subordinate science, and it is different precisely because in natural science matter is treated both as incumbent on form and as tailored for the form's proper expression. The natural scientist must therefore investigate matter up to the point in which its relations to form become clear (e.g., 194b10-13).

2.3 CONCLUSION (AND A LITTLE MORE)

Having outlined the general features of Aristotelian science, I've asked how to interpret Aristotle's proclamations regarding the character of disciplines in *Physics* II.2. I hope I have shown how the problem of establishing the interrelations of the *archai* of the study of nature—a problem which Aristotle promises will be given a "fresh start" in Book II (192b4)—drove Aristotle to invoke the subordinate sciences as an example, which in turn forced him to present additional considerations regarding matter and form. The nature of his statements about the disciplines are thus best understood as attempts to provide his reader with the conceptual machinery suitable for understanding how form can both be inseparable and prior to matter, and how form and matter can act as genuine sources of change. This conceptual machinery relied on the characterization of the subordinate, mixed-mathematical sciences as instances of what I've dubbed *tempered-hylomorphism*, a characterization that was borne out both from *Physics* II.2 (§2.2) and a consideration of the general features of Aristotelian science (§2.1).

Also in this chapter, I have considered the sources of unity, and by implication of distinctness, of branches of knowledge. This was done in order to prepare the way for the central claim regarding tempered-hylomorphism, but it is by no means of secondary interest.

In some sense, one can understand Aristotle's claims regarding the nature of the mixed-mathematical sciences as an attempt to account for their internal unity—their coherence as distinct fields of study—in face of the obvious fact that they seem to straddle disciplinary boundaries. I will continue this project of outlining ways in which a science can be said to be both unified onto itself and distinct from other sciences in the next chapter. There, I will characterize the attempts of some Renaissance Aristotelians to apply the conceptual machinery associated with the mixed-mathematical sciences to the problems of the unity of the whole of knowledge. The goal there will be to show that concepts whose home was in reflection on the mixed-mathematical sciences (i.e., those we studied in this chapter) were used in the Renaissance to analyze much broader features of the structure of the sciences. In particular, they were used to analyze the relationship between *metaphysics* and the remainder of the speculative sciences.

3.0 SUBALTERNATION, METAPHYSICS AND THE UNITY OF SCIENCE

In the previous chapter, I considered general features of Aristotelian science and what they might imply regarding the place of mixed-mathematics in the Aristotelian scientific landscape. I focused there on the notion of "tempered-hylomorphism"—the idea that a mixed-mathematical science treats material and mathematical considerations *jointly*, but in a way that falls short of the genuinely hylomorphic treatment demanded of a natural science. To prepare the ground for the discussion of tempered-hylomorphism, I also set forth ways in which a sciences can individuated or said to be one; that is, grounds on which the unity of a science can be attributed.

In the present chapter, I continue the focus on the individuation and unity of sciences, but confine my attention to the notion of "subalternation". My aim is to show that Renaissance scholastic-Aristotelianism (of a variety to be specified in §3.1) conceptualized the nature of the mixed-mathematical sciences through that notion and, moreover, used it in order to further conceptualize the interrelations of the speculative sciences. Put differently, I intend to show that a concept whose home was in the domain of the mathematical sciences was exported and its range of application expanded to include, most importantly, the relation of metaphysics to physics and mathematics. My evidence for this claim will come from considering a question that had become standard in Renaissance scholastic-Aristotelian writing; namely, "whether all sciences are subalternated to metaphysics". The focus on subalternation and metaphysics will return in later chapters, there in a Cartesian context.

It should be noted at the outset that the two thinkers whose opinion regarding the unity and subalternation of the sciences I am about to examine are chronologically some of the last in a long, tangled tradition. Questions on this topic in the context of first philosophy date back at least to Peter Lombard's *Sentences* (*circa* 1150), and the conceptual issues surrounding and prompting these questions date back (as the previous chapter demonstrated) to Aristotle. And so, although my examination of two thinkers certainly cannot guarantee that my thesis regarding the centrality of the concept of subalternation to the analysis of the structure of the sciences holds true generally, I believe the secondary literature on the variety of ancient and medieval sources supports my case strongly.[1] With that said, let me set out the reasons, historiographical and evidential, for my selection of Eustachius a Sacto Paolo and Francisco Suárez as figures of study.

3.1 SETTING THE SCENE

From a contemporary point of view, early-modern scholastic-Aristotelianism seems a vast tapestry of heterogeneous and often conflicting teachings. This outlook is so pervasive in contemporary scholarship that the older, opposite view—that scholastic-Aristotelianism was generally univocal—is customarily labeled ahistorical. And for good reason.

Nevertheless, the older view is not entirely without merit.[2] Even in early-modern times, the subtleties of Aristotelianism were ignored by those who were not themselves Aristotelians, and even by some who were. Descartes, a member of the first class, ordinarily portrayed scholastic-Aristotelianism as a uniform body of thought and was obvious enough in his homogenizing tendency that he was warned by Mersenne against doing so. In a letter dated 11 November 1640, for example, Descartes pronounced characteristically:

> I do not think that the diversity of the opinions of the scholastics makes their philosophy difficult to refute. It is easy to overturn the foundations on which they all agree, and once that has been done, all their disagreements over detail will seem foolish. (AT III 232, CSMK 156)

Galileo, Hobbes, and later Newton and Locke were equally indiscriminant is their criticisms of the Aristotelians.

[1] See, in no particular order, Weisheipl 1965, Kane 1954, Livesey and de Carlenis 1994, Aquinas and Maurer 1963, Laird 1983, Livesey and Lombard 1989, Livesey 1982, Steneck 1975, Livesey 1990, Laird 1987, Siraisi 1973, Edwards 1960: 97ff., Mikkeli 1999.

[2] I have also addressed this point in Chapter 1, footnote 3.

Despite the (correct) contemporary bias against taking Descartes' statements as representative of the actual state of Aristotelianism, Descartes' proclamations—as arrogant as they seem—were not merely artifacts of hubris. As has been noted by students of Aristotelianism, certain scholastic-Aristotelian writings—those commonly written for pedagogical purposes—portrayed Aristotelianism as more rigid and prone to orthodoxy than it truly was. As with contemporary textbooks, pedagogical purposes could take precedence over an accurate mapping of the intellectual landscape and distort the lay of the land.[3] The way in which Aristotelianism represented itself was not always as faithful as it might have been. For this reason, as Edwards (2007) notes, it is one of the challenges of contemporary scholarship to accurately depict Aristotelianism both as it actually was and as it was portrayed by its contemporaneous opponents *and* champions. Although the two goals may sometimes conflict with one another, each serves its proper historiographical purpose.

For the purposes of this chapter, I am concerned primarily with the way in which scholastic-Aristotelianism represented itself. Since one of my main goals in this dissertation is to come to a new understanding of Descartes, and since there is ample evidence to suggest that Descartes consulted scholastic-Aristotelian textbooks in order to form his understanding of scholastic-Aristotelianism (more on this in a second), I am concerned here with how scholastic-Aristotelianism was portrayed by the textbook genre. Put colloquially, I am interested in the sales-pitch of scholastic-Aristotelianism. Eustachius a Sancto Paolo and Francisco Suárez are useful for this end. I will use the first as my example of a purely pedagogical writer and the second as a way to flesh-out and justify the account of Eustachius a Sancto Paolo with a writer who, although not writing for pedagogical purposes, was taken as authoritative, particularly in the strain of scholastic-Aristotelian writings from which Eustachius' writings emerged. This 'strain' and the juxtaposition of Eustachius and Suárez will be discussed shortly. At any rate, I take this juxtaposition to show that the opinions I will examine were extant both in boiled-down summaries of scholastic-Aristotelianisms and in more reflective, nuanced works. But why these particular writers?

Eustachius a Sancto Paolo, although his place in contemporary scholarship overestimates his importance to his contemporaries, is particularly useful in evaluating Descartes'

[3] Reif 1969, Wallace 1988, and Schmitt 1988.

understanding of his own intellectual context. Although Descartes is usually reticent to name names, he explicitly commended Eustachius a Sancto Paolo's textbook, the *Summa philosophiae quaripartita*, as a lucid source for and précis of scholastic-Aristotelianism.[4] Immediately after the quote on page 47, he referred to it as "the best of its type ever made", representing those "foundations on which [all scholastics] agree".[5] Descartes also recalled that he had read Eustachius while a student at La Flèche (although he could not remember the author's name, only his monastic order) and conceived of creating a Cartesian commentary on Aristotelian philosophy using Eustachius' *Summa* as the basis of exposition.[6] For a while, Descartes attempted to find a version of orthodox Aristotelianism more suited to his task, but after entertaining another popular mid-century textbook—the *Totius philosophiae* of Abra de Raconis (1633)—became more confident in his original choice:

> I'ay vu la Philosophie de Monsieur de Raconis, mais elle est bien moins propre à mon dessein que celle du Pere Eustache; & pour les Conimbres, il sont trop longs; mais ie souhaiterois bien de bon coeur, qu'ils eussent écrit aussi briévement que l'autre, & i'aimerois bien mieux avoir affaire à la grande Societé, qu'à un particulier. (Letter to Mersenne, 3 December, 1640; AT III 251)[7]

Descartes' commentary project was ultimately abandoned for the project of the *Principles*.[8]

Descartes was not alone in regarding the *Summa* as an authoritative summary of scholastic philosophy. Jacques Du Roure, the author of the first textbook in Cartesian philosophy, also recommended Eustachius as a example of scholastic philosophy with which his readers ought to be familiar (*La Philosophie divisée en toutes ses parties*, 1654: 5). The number of

[4]The *Summa* was first printed in Paris in 1609 (then 1614, 1620, 1623, 1626, 1627) and Lyon (1620), Cologne (1620,1629), Geneva (1638, 1647), Cambridge (1640, 1648), and Leiden (1647). Eustachius, a French Cicstercian Professor of philosophy and theology at the Sorbonne, wrote several popular textbooks, including the *Summa theologiae tripartita* and the *Ethica sive Summa moralis disciplinae*, on which young Isaac Newton took still-surviving notes (Cambridge University Library, ms. add 3996 ff. 155v-116r, see also McGuire and Tamny 1983). For more on Eustachius, see Lejeune, Antoine de Saint-Pierre (1646), *Vie du R. P. Eustache de Saint-Paul Asseline*. Paris; Standaert, M (1961), 'Eustache de Saint-Paul Asseline', in Marcel Viller et al. (eds.) (1932-1995), *Dictionnaire de spiritualité ascetique et mystique* (17 tomes in 21 vols.), Paris: Beauchesne, t. 4, col. 1701-1705; Gilson, E. (1982), *Index scolastico-cartésien*, 2nd. ed., Paris: Alcan.

[5]Letter to Mersenne, 11 November 1640; AT III 232, CSMK 156.

[6]AT III 190, CSMK 154; AT III 232–233, CSMK 156–157; AT III 259–260, CSMK 161.

[7]Since no authoritative translation of this passage exists, I've left it in the original language. In this chapter, I will do the same for all foreign language quotes of which no authoritative translation exists.

[8]Descartes states that he had abandoned the original project due to Eustachius' death, but Roger Ariew has suggested that it is more likely that he decided against the project independently, see Ariew 1996.

printings which the *Summa* underwent—in France, Germany, Switzerland, and England—further attest to its standing in the mainstream as a fair representation of Aristotelianism.[9]

Not only in his own time, but in contemporary scholarship, Eustachius' is taken as representative of a mainstream, if watered down, version of mid-seventeenth century Jesuit-influenced Aristotelianism. Dennis Des Chene, for example, notes that the *Summa* was practically a "*Cliff's Notes*" of the Coimbra commentaries, providing "a good listing of many of the statements Aristotelians thought were true"(1996: 11). Eustachius' penchant for cribbing from these central Jesuit texts is what allows him to be juxtaposed with the Suárez, although Eustachius himself was not a Jesuit. Eustachius' relation to the Coimbrans also makes him doubly useful as a reference point for understanding Descartes, since the Coimbra commentaries were used by the Jesuits at La Flèche during Descartes' education and, like Eustachius' *Summa*, were remembered by him years later.[10]

Francisco Suárez, in contradistinction to Eustachius, is anything but watered-down. A Spanish Jesuit who taught both at the University of Coimbra and the Collegio Romano (among others), his *Disputationes Metaphysicae* (1597-) is considered an "encyclopedia of Scholasticism" (Vazques-Amaral 1984). Its importance, however, extends far beyond its wide scope. With the *Disputationes Metaphysicae*, Suárez formed a watershed for scholasticism: before the *Disputationes*, works on metaphysics almost always took the form of a commentary on Aristotle's original text; after it, works on metaphysics (particularly general metaphysics) were almost always free-standing or parts of the newly popular *cursus* of philosophy, but no longer mere commentaries.[11] Although I make no use of this fact in my analysis of Suárez, it demonstrates that the *Disputationes* were widely read and influential. In fact, Suárez's metaphysical views and novel argumentation became something of a standard of late sixteenth- and seventeenth-century philosophy and impacted both erudite opinion and popular expositions. There is also evidence to show that many of the *novatores* of the seventeenth-century (e.g., Descartes, Hobbes, Leibniz, and Spinoza) read Suárez directly.[12]

Jointly, Eustachius and Suárez offer a balanced picture of what Stephen Menn (1997: 226)

[9] See footnote 4 for Eustachius' publication list.

[10] Letter to Mersenne, 28 October 1620; AT III 190, CSMK 154.

[11] See Copleston 1968: 353–355, Zubiri 1942: 128. Of course, Suárez was also following the lead of Fonseca and Toletus, but his influence seems to have been farther reaching.

[12] See Des Chene 1996: 7-11, Leijenhorst 2002: 10, Hattab 1998: 39-41.

has called "liberal Jesuit scholasticism" (although, of course, Eustachius was not a Jesuit). Although Menn (2000) also expressed regret about the label, there is no doubt it captured an important strain of scholastic-Aristotelianism, albeit one that is mostly devoid of dedicated Ockamists, Averroists, Scotists and Thomists. Despite this, this strain is influential, and crucial for understanding Descartes.[13] What I will focus on in the next pages is how the representatives of liberal Jesuit scholasticism understood the unity of science, subalternation, and their relation to metaphysics.

3.2 UNITY, SUBALTERNATION, AND METAPHYSICS IN EUSTACHIUS

Eustachius, like many textbook authors, discusses the unity and subalternation of the sciences in the first division of the four traditional divisions of philosophy, namely "Logic" or "Dialectics", at the end of a section on the demonstrative syllogism (*de Syllogsmo demonstrativo*).[14] The text is provided in Appendix A.

Eustachius begins his discussion by making clear that the attribution of unity to a science must involve all 'elements' that make up the science: its definitions, its principles, as well as the syllogisms that make up the body of the science itself. Because Eustachius' discussion in the earlier part of the treatise had focused on the demonstrative syllogism, and because the attainment of this sort of syllogism is what raises a body of speculation to the status of *a science*, Eustachius wants to make clear that attributions of *unity* to a science must apply to more than just collections of demonstrative syllogisms. Attributions of unity must apply to the wider range of conceptual elements that are involved in the very formulation of demonstrative syllogisms.[15] However, because of this requirement, the attribution of unity to a science is of limited force.

[13] Both Eustachius and Suárez have been used extensively in the secondary literature to contrast scholastic-Aristotelian opinions with those of Descartes. See, for just a few examples, Hattab 2007, Armour 1993, Ariew and Grene 1995, Lennon 1995, Pitte 1988, Doig 1977, Vazques-Amaral 1984, Kambouchner 2005.

[14] The four parts of Eustachius' *Summa*, as well as many other philosophical textbooks, are Logic or Dialectics, Ethics, Physics, and Metaphysics. Eustachius also brings up the question in the part on metaphysics, but refers to the discussion in the part on "Logica seu Dialectica" as the authoritative one. On traditional textbook constitution, see Garber and Ayers 1998: Ch. 4.

[15] *Summa*, Part I: 237.

Science, Eustachius claims, is unified—or literally, can be said to be one—only by aggregation; that is, in the same way in which an army, a nation, or a gathering of men can be said to be one: "[Scientia] nec aliter una dici potest, quam exercitus, respublica, coetus hominum unus dicitur" (*Summa*, Part I: 237). Although *some* principle of unity can be found in such instances—an army is gathered for the purpose of fighting an enemy, a nation is formed around some political, cultural, or geographical idea—this principle cannot render the object under study an *individual*, that is, a numerically single entity. Put differently, the principle of unity in the case of whole sciences cannot support properly constituted composites of matter and form—something true unities would require. This introductory note is rather important, since it sets interpretive boundaries on the rest of Eustachius' discussion. Before he even articulates the finer points of the question of unity, Eustachius makes clear that the subject of the unity of *science* does not contain what a reader might expect from a discussion of unity and individuation of *things*, itself a subject with a long, tangled history.[16] At any rate, this caveat will become important again shortly, when it will allow Eustachius to hold that a kind of unity can be found even between whole sciences that, taken by themselves, seem to be distinct.

Having made this introductory caveat, Eustachius asks on what grounds we are licensed to attribute unity to a body of knowledge.[17]

> Secundo quaeritur undenam petatur unitas & distinctio eiusmodi totalium scientiarun, verbi gratia, unde habeat Physica quod sit ac dicatur una scientia ab aliis distincta. (*Summa*, Part 1: 236)

His answer is that the primary grounds of attribution are the "total object" (*obiecto totali*) treated by the science. Eustachius speaks of the "total object" in two ways. First, he uses the phrase to describe the *uber*-object created by taking under consideration all objects treated by a science, as if in a sum.[18] Second, he uses the phrase merely as a way of speaking about

[16]On the unity and individuation of *res* in the scholastic literature, see Kretzmann et al. 1982 and Gracia 1994. On Eustachius in particular, see Lennon 1995: 177–179.

[17]In truth, nestled between the introductory caveat and the question quoted above, Eustachius introduces, almost in bullet-point form, some considerations regarding the "total object" of a unified science. These considerations seem unnecessary to repeat here, since Eustachius reiterates them in the subsequent paragraphs.

[18]In contemporary mereological theory, the idea that any individuals have a mereological sum that constitutes an object is called "collectivism". See Varzi 2006

the genus treated by the science, as a single entity. Given Eustachius' first use, perhaps a better translation of the phrase would have been 'the entirety of objects'. However, this perverts the singular "obiectum", which Eustachius uses consistently, and the moral of the entire discussion—a moral that is clear despite the double connotation of the phrase "obiecto totali". The moral is that when considering the unity of a body of knowledge we need to be concerned with the genus with which the science is concerned, taken as a whole. Eustachius also calls this the "subject" of the science (e.g., *Summa* 240). Put technically, Eustachius endorses the *Objects Criterion* discussed in the previous chapter.

Eustachius does not leave this moral and the equality of total object and genus implicit. After claiming that unity is attributed on the basis of the total object, he provides the following example:

> Sic Dialectica dicitur una totalis scientia, quia tota versatur circa *unum genus obiectu totale*, nempe circa actionem mentis dirigibilem iuxta optimum cognoscendi modum, quod quidem in ratione obiecti differt a quovis alterius scientiae obiecto, unde & Dialectica differt a quavis alia totali scientia (*Summa*, Part 1: 236, emphasis added)

Dialectics is "one, total science", he writes, because it revolves around a type of action of the mind, a type of action that comprises a genus or total object. That type of action is directed towards optimal understanding—a task Dialectics is supposed to aid in attaining—and thus for Eustachius Dialectics is counted as a *single* science. As further exemplification of the *Object Criterion*, Eustachius invokes the familiar example of the three theoretical sciences (see §2.1.2). He holds that each has a total object—or encompasses a subject genus—that is defined by a different level of abstraction from particulars.[19] Physics, he claims, abstracts only from the *particularity* of particulars—their "individuating condition"—but nevertheless treats objects as concrete, enmattered entities. Although it was a common (but not an unobjectionable) claim in scholastic metaphysics that matter is ultimately responsible for the individuation of substances, Physics, according to Eustachius, does not abstract from matter *entirely*, but only from the individuating effect of matter; that is, from particulars or individuals.[20] Other features of a substance that are incumbent on its being en-mattered

[19]Eustachius further discusses this in *Summa*, Part 1: 155–157, Part 3: 1–4, Part. 4: 1-3. See also Hatfield 1985: footnote 4.

[20]For causes of individuation and the question of precisely what (matter, Scotist haecceity, etc.) physics must abstract from in order to abstract from particulars, see Gracia 1994, Barber and Gracia 1994 and

are still considered by Physics. Mathematics, on the other hand, abstracts from all matter and thus from all sensible qualities found in concrete objects. Metaphysics, finally, abstracts from both sensible matter and from quantity.[21] Eustachius' further articulates the thesis which the three-fold schema is intended to exemplify with the finer-grained example of mathematics. This passage, although lengthy, is worth quoting is full here, since it harks back to the central passage of Chapter 2 and the question regarding the proper business of the mixed-mathematician studied there.

> Nam Physicus considerat quidem entia naturalia in universum abstracta a singularibus, non vero abstracta a qualitate sensibili, aut quanitate; deinde Mathematica, cuius obiectum, nempe ens Mathematicum, ab omni *ferme* materia sensibili non tantum a singulari abstractum est: Mathematicus enim considerat quantitatem vel puram, id est, omni qualitate sensibili distitutam, ut in Arithmetica & Geometria; vel cum una aut altera duntaxat specie qualitatis coniunctam, ut in musica, quanitatem cum sono, in Astrologia quantitatem cum motu coelesti, in Optica quantitatem cum lumine vel coloe; denique Metaphysica, cuius obiectum nempe ens, ut ens, abstrahitur ab omni materia non tant'um singulari aut sensibili, sed etiam ab intelligibili, ut patet. (*Summa*, Part 1: 237, emphasis added)

As Eustachius states, mathematicians come in two flavors: first, there is the sort of mathematician who studies pure quantity—quantity "divorced from all sensible quality"—and, second, there is the sort of mathematician studies quantity conjoined "to some extent with one or another kind of quality", a mathematician who abstract from *nearly* ("ferme") all matter. This second type of mathematician is the mixed mathematician; that is, one who studies quantity conjoined with sound (as in harmonics), quantity conjoined with vision (as in optics), or quantity conjoined with celestial motion (as in astronomy).

Two points need to be made regarding this interpretation of the mixed mathematician in relation to the question of unity. First, it is plain that according to this interpretation the so-called mixed-mathematical sciences are types of mathematics—a fact I will highlight when drawing overall conclusions from Eustachius' text. That these sciences are branches of mathematics is clear in the above passage because they share with mathematics the focus on *quantity*, continuous or discrete. Since neither physics nor metaphysics is similarly concerned with quantity, it seems that the only division within this disciplinary classification to which

references in Cross 2003.

[21] Eustachius uses an object's extension or "intelligible matter" as the hallmark of quantity. For the sources and history of the concept of "intelligible matter "see Anderson 1969a,b and Annas 1976: 30ff..

optics, harmonics and astronomy can belong is mathematics. Eustachius believes that this is an important point and calls the reader's attention to it immediately after the passage above:

> Ubi aduerte[!], quoniam in mathematicis non unus plane est abstractionis modus a materia sensibili, ideo penes diversum illum abstractionis modum diversa in illis esse totalium scientiarum genera, quod non contingit in physicis, aut metaphysicis. (*Summa*, Part 1: 237–238)

Eustachius emphasizes here that there can be different sciences *within* mathematics, since mathematical abstraction can take different forms. In other words, since mathematical abstraction can separate or not separate quantity from different sorts of sensible qualities *and still remain mathematical*, the different sciences that result from these different abstractions all remain *mathematical* sciences.[22]

The second point is that Eustachius' discussion of the unity of science by way of the "total object" of the various theoretical sciences implies that what in the last chapter we called the *methods criterion* is co-extensive with the *objects criterion.* In fact, it seems that according to Eustachius the *methods criterion* explains why the *objects criterion* holds as an individuating principle. Explicitly, metaphysics, physics, and mathematics are distinguished by their "total objects" because human cognizers apply different sorts of abstraction in order to generate their "total objects". Since the process of human cognition determines the types of objects with which particular sciences are concerned, it is human cognition that is ultimately responsible for the individuation of the sciences (given the constraints provided by the ways in which things actually exist in the world).[23]

After having individuated sciences according to their genera, Eustachius considers their order, that is, their relations to one another. Eustachius first lists four types of order according to which these relations can be considered (the order of nature, the order of dignity, the order of teaching, and the order of time) and determines which sciences are

[22]See also *Summa*, Part 1: 241–242. The examples there concern the subalternation of astronomy/astrology to those parts of the physical sciences that deal with the heavens, and the overall subalternation of geometry to physics.

[23]Although I will not analyze the equality of the *objects* and *methods criteria* in Suárez, a similar case can be made there. There, the equality of criteria is established by the fact that the *habitus* of a science—something akin to the cognitive practices that must be 'had' in order to practice the science—is ultimately responsible for the generation of its objects. See *Disputationes Metaphysicae*, Disp. I, Sec. II, III and Disp. 55 for an extended treatment of "habitus".

prior and which *posterior* according to the four orders.[24] This discussion is rather telegraphic and need not be repeated fully here. However, in considering the order of *nature*–the only ontologically-based category in the four orders—Eustachius slips from talking about priory and posteriority to talking about subalternation, inferiority, and superiority. He writes that:

> Ordine naturae Metaphysica antecedit, sequitur Physica, tandem Mathematicae, eoquod obiectum Metaphysicae natura est omnium primum ut pote universalissimum; obiectum vero Physicae natura etiam prius est obiecto Mathematicarum, quia est communius: Adde Mathematicas esse inferiores, & subalternas Physicae, ut pote a qua sua principia ut plurimum desumunt. (*Summa*, Part I: 239)

As this passage makes clear, the order of nature is determined by the hierarchy of "total objects" treated by particular sciences. Explicitly, a science that is prior in the order of nature concerns a more general "total object" than a science that is posterior in that order. Metaphysics is prior to physics because its total object is most general—in fact, it is "universal". Given what we have already said about the individuation of sciences in Eustachius, it is clear that for Eustachius, the *objects criterion* guides both the individuation and ordering of sciences. Whether the criterion can properly serve such double-duty is yet to be determined, but note that Eustachius bolsters it in the above passage with the *principles criterion*. In explicating the posteriority of mathematics to physics, Eustachius adds the claim that mathematics follows physics not only because of its object, but because it "picks out" (*desumunt*) its principles from those of physics. It is at this point that he invokes subalternation to characterize the relationship of the two sciences (albeit without yet defining the idea).

The explicit discussion of subalternation and its relation to the objects criterion comes at the close of the disputation. Curiously, Eustachius begins with a word of caution to those who are prone to terminological confusion.

> Ut autem appellant subalternatam, inferiorem vero subalternantem, impropriam eorum locutionem praetermittam qui superiorem scientiam dicitur in genere scientia alteri subalternata que illi subest seu subiicitur, atque ab ea dependet:50 illa autem subiectio seu dependentia dicitur subalternatio.

Ironically, while this passage intends to defuse terminological confusion, it is so grammatically muddled in the *Summa* of 1609 (the edition I have been using) that it is unclear precisely

[24] *Summa*, Part I: 238.

what terminological confusion it intends to defuse. In a later edition, the passage is altered to read:

> Ut autem improptiam eorum locutionem pretermittam qui superiorem scientiam appellant subalternatam, inferiorem vero subalternantem; dicitur in genere scientia alteri subalternata, quae illi subest seu subjicitur, atque ab ea dependent: illa autem subjectio seu dependentia dicitur subalternatio. (*Summa philosophiae quaripartita* 1648: 140)

As the later passage disambiguates, the confusion concerns the ascription of superiority and inferiority to subalternate and subalternating sciences. Before he even provides a definition of subalternation, Eustachius alerts those who believe that subalternated sciences are *superior* to their subalternating sciences that they are using an "improper locution". Since I am not aware of any scholastic commentator who had previously mixed terms so flagrantly, I believe that Eustachius' note of caution is simply meant as a facile introduction to his definition of subalternation or, perhaps, as a connective between the discussion of order and the proper definition itself.

At any rate, the definition which follows states that "a science is subalternate to another, [when] it is below the other or subjugated to it, as well as dependent on it [literally: hangs down from it]. This subjugation or dependence is called subalternation".[25] Taken as it stands, the definition suggests that subalternation is a binary, asymmetrical relation. That is, it holds between pairs of sciences and is not mutual. Furthermore, the notion of "dependence" suggests that the subalternating sciences is *sine qua non* for the subalternated science: if one science depends on another, then the removal of the latter would in effect destroy the former. What *in particular* depends on what and what is subjugated to what (and how) is left unspecified at this point.

Eustachius also does not further specify what he means by "dependence" or "subjugation" directly, but offers additional distinctions as a means of clarification. First, he notes that a science can be subalternated to another either partially or totally, with total subalternation being the "proper" sort of subalternation.[26] In partial subalternation, the subalternate science depends on or "follows"—a new phrase—only part of the superior science. In total subalternation, the subalternate science depends on or "follows" the whole

[25] *Summa*, Part 1: 240.
[26] *Summa*, Part 1: 240.

of the subalternating science. Of course, this is still terribly vague, but at this point in the disputation, Eustachius has not told us what dependence, subjugation, or following actually mean! At any rate, although the idea that total subalternation is "proper" is interesting in itself, since we are concerned here with the definition, what is more interesting is the addition of the notion that a subalternate science "follows" (from *secere*) the subalternating science. As in English, *secundum* can be taken in two senses: It can be taken to indicate a spatial metaphor and thus signify 'coming after' or 'coming in succession'. But it can also be taken to signify agreement and concordance. Taken in this second sense, the claim that a subalternate science 'follows' means not only that it *depends* on a subalternating science (i.e., that the subalternating is a *sine qua non*), but that, in some sense, it *accords* with that science, that its content is relevantly harmonious.

Eustachius' second clarificatory distinction concerns the causes of subalternation and explicates which parts of a science *in particular* are subalternated to one another. The subalternation of one science to another, he writes, can be due to three causes: its goals, its objects, and its principles. First, there is the subalternation of goals. Eustachius writes:

> [F]inis Dialecticae, qui est mentis operationes ad legitimum cognoscendi modum dirigere, refertur ad finem theoreticarum scientiarum nempe veritatis contemplationem, ideoque Dialectica caeteris scientiis tanquam administra subalternatur. (*Summa*, Part I: 240)

The goal of Dialectics, Eustachius claims, "is referred to" the goal of the theoretical sciences, which is the contemplation of truth. In this way, dialectics, which is not a theoretical science, is a servant (*administra*) of the theoretical sciences: its own goal it to aid in the attainment of the goal of the theoretical sciences. Note that this subalternation of goals has no implications for the content of the subalternated science *per se*, only for the end to which it is exercised. Eustachius does not elaborate further on the subalternation of goals, but the imagery of the servant fits well with the explication of subalternation as a type of subjugation: the servant derives neither its existence nor character from its master; however, the servant's function is to serve its master and is thus subjugated to it.

Second, subalternation can be due to the object of the subalternated science:

> Secundo subalternatio contingit ratione subiecti seu obiecti, cum nempe subiectum seu obiectum unius subalternis obiecte continetur (Summa, 240–1).

Eustachius holds that one science is subalternated to another when the object of the former is 'contained' in the object of the latter. The central idea here, which should be familiar from the previous chapter and from Eustachius' discussion of unity, is that a subalternate science treats a sub-class of the objects with which the subalternating science deals. This way of being subalternated captures well the *sine qua non* relationship between the subalternating and the subalternated sciences: without the subalternating science, there would be no object for the subalternate science to study. However, there are two ways to read this claim regarding the object of the subalternate science, ways that have not been sufficiently distinguished thus far. On an 'extensional' reading, the subalternated science treats a numerical subset of the particulars studied by the subalternating science. On an 'intensional' reading, the subalternated science treats an object that is somehow (in a way to be specified shortly) an intensional component of the object treated by the subalternating science, with no implications regarding the enumeration of particulars. As the following example makes clear, Eustachius' reading is an intensional one:

> sic Astrologia subalternatur ei parti Physiologiae, quaede coelo disserit; Physici enim est in ea parte non tantum de substantia coelesti, sed etiam de ipsius quantitate & motu dissere, que etiam ex instituto contemplatur Astrologus.

Astrology/Astronomy, Eustachius claims, is subalternated to physics because it treats a subset of what physics is able to treat. The claim, however, is not that physics treats both celestial and terrestrial objects whereas astrology/astronomy treats only celestial ones. Rather, the claim is that whereas physics can treat the substance, quantity, and motion of the heavens, astrology/astrology treats only heavenly quantity and motion. This usage fits well Eustachius' original use of the *objects criterion* to individuate sciences: by an 'object of a science' he means not a real particular, but a *type* of object treated by the science.

Third, subalternation depends on principles:

> Tertio denique subalternatio contingit raione principiorum, cum nempe principia inferioris scientiae a superiori scientiae desumuntur, itaut principia illa inferioris, sint conclusiones superioris[.] (*Summa*, d241).

This idea should also be familiar from the previous chapter—it was one of the senses of subalternation for Aristotle. Subalternation occurs, Eustachius claims, when principles of a higher science are taken over (*desumuntur*, also 'chosen' or 'picked out') by a lower science. In the lower science, those principles are presumed to be true and are thus taken as indemonstrable, but in the higher science, they are themselves conclusions of demonstrations. Note that this is perhaps the most full-blooded sense of subalternation discussed by Eustachius, since it serves to determine not only the very existence of the subalternate science (i.e., the higher science is still a *sine qua non*), but its content as well: the principles borrowed from the higher science provide the lower science with the concepts (as well as the inferential rules by which they relate to one another) by which to analyze its subject matter. Without these, there is nothing that the lower is *about* (in the way that mathematics is *about* quantity, medicine *about* health, etc).[27] To put the point using Eustachius' metaphorical language, on this version of subalternation the lower science is not only subjugated and dependent on the higher science, but truly accords with, or "follows" it.[28]

Having explained the meaning of "subalternation", we should now examine its application to the unity of science vis-à-vis metaphysics. Oddly, although the entire disputation builds up to this application, it occupies only a single sentence, the disputation's last. It reads:

> Denique omnes scientiae Metaphysics, licet non omnino proprie aliqua tamen ratione subalternari, tum quia Metaphysica continet generalissima quaedam principia, quibus principia caeterarum scientiarum saltem ductu ad incommodum confirmari possunt; tum quia subiectum Metaphysicae; quod est ens in suo conceptu, saltem confuse caeterarum omnium scientiarum subiecta continet. (Summa, Part I: 242)

Eustachius holds that all sciences are subalternated to metaphysics. He provides two reasons. First of all, the objects of all sciences are contained, at least "in a confused way", in the object of metaphysics. That is, because the object of metaphysics is being *qua* being,[29] and since the other sciences consider types of being, the objects of the other sciences are contained in metaphysics. Put differently, from the vantage point of metaphysics, the other sciences consider being *qua* being with the addition of some conditions. This is precisely

[27]Perhaps it is unfair to call this sort of aboutness "content", since it includes some features of a science that are commonly considered "formal", like inferential rules. However, this overall set of features determines the way in which a science treats its subject matter, and so, even on a more strict reading of "content", there would be no content to a science if it was not for this overall set of features.

[28]This also allows us to better understand what Eustachius might have meant by "total" and "partial" subalternation. A case of "total" subalternation is one in which a subalternate science picks out *all* its principles from the subalternating science; a case of "partial" subalternation is one in which a subalternate science picks out only *some* of its principles from the subalternating science.

[29]See *Summa*, Part IV: 1-3.

how Eustachius defined the relationship between the objects of the mixed-mathematical science and the objects of pure mathematics. Second, the principles of all other sciences can be confirmed by metaphysics. Although Eustachius does not provide actual examples for this, his belief is clear: metaphysics could provide demonstrations whose conclusions would furnish all other sciences with their principles.

3.3 IMPLICATIONS AND CONCLUSIONS

This intricate conception requires several comments.

First, since the importance of *objects criterion* has come up again and again, I should note that its role is rather multifarious. On the one hand, Eustachius had initially used the criterion in order to individuate sciences. Thus, metaphysics, physics, and mathematics constituted different sciences because their objects were taken to be different. At the same time, however, Eustachius used the criterion in order to determine the direction of the subalternating relation. Thus, he held that metaphysics contains, at least in "a confused way", the subject of all other sciences because all other sciences study some type of being. The relations between the objects of the various sciences thus determine both the possible unity and possible subalternation of those sciences. Focusing on metaphysics, we can see that claims to unity and claims to subalternation are not mutually exclusive. In fact, if we were to straddle Eustachius with a single view—one which he does not admit explicitly, but one which seems plausible given his other statements—we might claim that the speculative sciences constitute a unity, in a way, but also constitute a structure of pair-wise subalternations, depending on the way in which their objects are conceived in relation to one another. In other words, using the *objects criterion* to individuate sciences, we may say that all sciences are unified under the banner of metaphysics (since, loosely speaking, metaphysics contains the objects of all the speculative sciences), but we may also say that the sciences are distinct and related to one another by way of the subalternation relation (since, strictly speaking, each speculative science treats a different type of object). The way we conceive the variety of objects studied by these sciences will determine which of these two position we take. Of course, the unity mentioned here is rather loose (since metaphysics encompasses the objects of the other sciences only in "a confused way"), but this does fit with Eustachius' initial statement that the type of unity appropriate for sciences is rather loose, one that is on par with the unity of a state or an army. Still, the position that the speculative sciences constitute both a loose type of unity and a structure of subalternate relation is an interesting one. In the following section, we shall see that Suárez is much clearer about the way in which the objects of various sciences must be conceived in relation to one another, and thus much clearer about the ways in which metaphysics might be related to the other speculative sciences.

Second, we should relate what we have found in Eustachius to the findings of the previous chapter. In the previous chapter, I endorsed "tempered-hylomorphism" as a way of characterizing what is unique to the mixed-mathematical, subalternate sciences. I defined tempered-hylomorphism there as the idea that the mixed-mathematical sciences treat material and mathematical considerations *jointly*. From Eustachius' discussion of subalternation, we've learned that subalternation can take place not only between pure- and mixed- mathematics, but between a variety of sciences, and in a variety of ways. Consequently, although tempered-hylomorphism remains an accurate representation of the mixed-*mathematical* sciences, it is no longer applicable in a more general context. This suggest that tempered-hylomorphism and the subalternation relation should not be understood as two sides of a single coin: this equivalence may be true in the strict context of the mixed-mathematical sciences, but not otherwise, say, in the case of the subalternation of Dialectics to the speculative sciences. This leads to our third point.

Third, and most importantly, I'd like to point to the wide scope of Eustachius' concept of subalternation. This is the central point of the present chapter. *Eustachius uses a concept—"subalternation"—borrowed from reflection on the mathematical sciences and expands its scope to explicate the relationship between a variety of sciences, and particularly, the relationship between metaphysics and the remainder of the speculative sciences.* As noted earlier, there is no doubt that Eustachius considers optics, astronomy and harmonics to be branches of mathematics. There is also no doubt that he considers these to offer the clearest examples of subalternation. Most of his examples in this disputation are, in fact, taken from either optics or astronomy. Still, he finds no problem with expanding the scope of the concept and analyzing the relationship between other sciences in terms of subalternation, particularly the relationship between metaphysics and the speculative sciences. Given that Eustachius was an orthodox, pedagogical expositor of scholastic-Aristotelianism, there is very good reason to believe that this use of subalternation was not at all unusual. Perhaps, then, there is also reason to believe that when Descartes claimed that all his sciences follow from and depend on his metaphysics, his claim would not have seemed nearly as revolutionary as it first appears. In fact, since Descartes took himself to be producing a textbook to replace the likes of Eustachius' *Summa*, it is not entirely implausible to think that he may have endorsed this picture of the sciences, at least in part, in order to fit some of the expectations of his hoped-for scholastic audience.[30]

At any rate, in order to provide further support for this last claim regarding the general scope of "subalternation", it is now time to see whether the relation of metaphysics to other sciences was conceived in a similar way by other thinkers.

3.4 FURTHER EVIDENCE: SUBALTERNATION AND METAPHYSICS IN SUÁREZ

In this section I'll briefly examine Suárez's statements on the subject of metaphysics and subalternation in the *Disputationes Metaphysicae*, merely to confirm the conclusion reached above.

In the *Disputationes Metaphysicae* Suárez covers a range of topics similar to the ones reviewed in the previous section, but in relation to metaphysics alone.[31] In the first disputation ("About the Nature of First Philosophy or Metaphysics"), he determines the object and extension of metaphysics (Section I, II), considers whether metaphysics is a single science (Section III), considers its functions and ends (Section IV), whether it is the most perfect speculative science (Section V), and whether it is the most sought of the sciences (Section

[30]Roger Ariew argues for precisely this claim in Ariew 1992.

[31]A significant difference between the first part of the *Summa* and the *Disputationes Metaphysicae* is that the first is a work on logic, while the latter focuses on metaphysics. Consequently, Eustachius' scope is somewhat wider than Suárez's.

VI). I will focus here on Section V. In Section V, Suárez entertains the order of the sciences according to their wisdom, certitude, the perfection of their "habitus", their pedagogical worth, and the causes they study, only to affirm, rather conclusively, that "metaphysics is easily the first of all other sciences".[32] Like Eustachius, it is in this context that Suárez raises the question of subalternation in order to explicate the relation between metaphysics and the other sciences (having already established that metaphysics is "facile princeps" (*Disputationes Metaphysicae*, Sec. 1, Art. 44)). He offers six articles (46–52) under the title "Expeditur dubium de subalternatione aliarum scientiarum ad metaphysicam": *The doubt concerning the subalternation of the other sciences to metaphysics addressed.*

Suárez defines subalternation in the following way:

> [S]upponamus proprie scientiam illam dici subalternatam alteri, quae essentialiter seu necessario ex natura rei ab illa pendet in esse scientiae, ita ut esse scientia non possit nisi scientiae subalternanti coniungatur, et ab illa sumat evidentiam principiorum (*Disputationes Metaphysicae*, Part I, Sec. V. Art. 47).

In a case of subalternation, Suárez holds, a subalternated science is dependent for its very being on the subalternating science. More explicitly, a subalternated science would not be a science unless it was united to its subalternating science and through this union draw evidence for its principles. Suárez begins his discussion of subalternation where Eustachius ended: with a full-fledged notion of subalternation that suggests that the subalternate science depends on the subalternating science not only for its existence, but for its content (by way of its principles); that is, for what makes it what it is. He also makes clear, however, that this subalternation can be partial:

> Contingit vero interdum scientiam aliquam non in omnibus suis principiis, neque in omnium conclusionum demonstrationibus, sed in quibusdam habere dictam dependentiam a scientia superiori; et tunc dicitur illi subalternata, non in totum, sed ex parte, seu partiali subordinatione, non totali (*Disputationes Metaphysicae*, Part I, Sec. V. Art. 48).

In other words, there are cases in which only some of the principles of a subalternated science are drawn from a subalternating science. The remainder of the subalternated science's principles must come from elsewhere.

[32] *Disputationes Metaphysicae*, Disp. 1, Sec. V, Art. 44.

Like Eustachius, Suárez also accounts for the subalternation of principles by reference to the objects of the sciences in question:

> Oriri autem solet haec dependentia unius scientiae ab alia ex subordinatione obiectorum; nam, sicut esse scientiae consistit in ordine ad obiectum, ita et principia sunt proportionata illi. Quapropter si obiecta duarum scientiarum non sunt inter se subordinata, utpote si sint genera vel species omnino condivisae, inter illas scientias non potest esse subalternatio (*Disputationes Metaphysicae*, Part I, Sec. V. Art. 49).

Because the principles that make up a science must be suited ("proportional") to the object with which the science is concerned ("the essence of a science consists in its relation to its object"), the subalternation of principles ultimately arises from the subordination of objects. If such subordination does not occur, as in the case of "completely separated" genera or species, subalternation cannot occur. In fact, subalternation can only occur when the object of two sciences are relevantly similar:

> Oportet ergo ut haec subalternatio in obiectis fundetur, nimirum in eo quod obiectum unius est idem cum obiecto alterius, adiuncta aliqua differentia accidentali, quae in esse entis sit per accidens, in esse autem scibilis sit aliquo modo per se, et constituat speciale obiectum scibile (*Disputationes Metaphysicae*, Part I, Sec. V. Art. 49).

It is here that Suárez's clarity as a philosopher comes to the fore. Whereas Eustachius left the relationship of the objects of two subalternating science vague, claiming only that one must be contained in the other, Suárez gives us precise criteria. The objects of a subalternating and a subalternated science must be the same, but with the addition of an accidental difference. This notion leads to a crucial problem: Sciences are supposed to treat what is essential to their objects, not what is accidentally true of them (see Chap. 2). If so, wouldn't a pair of sciences whose objects are separated only by an accidental difference be one and the same science? The accidental difference would be 'filtered-out' by the fact that the sciences treat only what is essential!

To address this problem, Suárez draws a distinction between an object as it is in itself and as it is *qua* the object of a science. Although, Suárez writes, in their very being (*esse*) the objects of a subalternating and subalternated science differ only accidentally, in their being known-objects (*obiectum scibile*), the objects differs essentially, and thus can be objects of two sciences, each of which studies its object *per se* (*Disputationes Metaphysicae*, Part I, Sec. V. Art. 49). It is in this way that the standard examples of subalternation must be understood:

> Ad subalternationem ergo absolutam et totalem, necesse est ut subiectum subalternatae scientiae addat accidentalem differentiam subiecto scientiae subalternantis, ut linea visualis addit lineae, numerus sonorus numero. . . (*Disputationes Metaphysicae*, Part I, Sec. V. Art. 50)

Suárez writes that for "total" subalternation, the object of the subalternated science must differ by an accidental difference from the object of the subalternating science, as "visual line" differs from "line" in optics and geometry or as "sonorous number" differs from number in harmonics and arithmetic. Although Suárez does not argue for the following claim, after the above passage he states that subalternation occurs in these cases *both* because the subalternated science considers a more complex whole, while the subalternating science abstract from it, *and* because the principles of the subalternated science are established by the subalternating science.

Given this conception of subalternation, Suárez holds that metaphysics does *not* subalternate any other science. In the first place, no other science wholly depends on metaphysics for its being, since no other science must derive the evidence for its principles from metaphysics. In fact, Suárez notes, the principles of any other science are immediately knowable through themselves because they become known through the set of cognitive practices (the *habitus*) that characterize that science. Metaphysics itself is not necessary. Interestingly, Suárez holds that metaphysics could, in principle, prove the principles of any other science, but such a proof would not be *necessary* to establish those principles. Moreover, such a proof would not be *a priori*: metaphysics can only prove the principles of other sciences by a *reductio* or by an "extrinsic cause" (and thus not *per se*), a mode of proof different than the one required by sciences themselves:[33]

> Etenim, licet metaphysica demonstrare possit aliquo modo illa principia, illa tamen demonstratio non est simpliciter necessaria ad iudicium evidens talium principiorum, cum ex terminis possint evidenter cognosci, et illa demonstratio non sit proprie a priori, sed per deductionem ad impossibile, vel ad summum per aliquam extrinsecam causam (*Disputationes Metaphysicae*, Part I, Sec. V. Art. 50).

[33] An admission: I an not clear what this "extrinsic cause" is supposed to be.

Metaphysics also does not subalternate the other sciences because the objects of the other sciences do not add an accident to the object of metaphysics. This difficult passage is worth quoting in full:

> Et ratio est, quia sub ente nihil continetur per accidens, sed per se; quod si sit aliqua scientia quae agit de aliquo ente rationis, illa nullo modo subordinatur metaphysicae quatenus agit de ente reali, quia ens rationis ut sic non continetur sub ente reali, sed est primo diversum; quatenus vero metaphysica agit de ente rationis, sic quodlibet ens rationis non per accidens sed per se continetur sub ente rationis ut sic, quod metaphysicus considerat; non ergo intercedit propria et totalis subalternatio (*Disputationes Metaphysicae*, Part I, Sec. V. Art. 51).

Suárez notes that the object of metaphysics is "being or substance" and the objects of the other sciences, in so far as they are real beings, are essentially contained in this object, not different from it accidentally. Although Suárez does not put it in the following terms, the thought here is that the object of the other sciences are related to those of metaphysics as genus and species, and so do not cross-genera (and thus add an *accidental* difference) in the way required for subalternation. Moreover, in so far as the objects of the other sciences are *not* real beings, that is, insofar as they are considered merely as *being of reason* (*ens rationis*, what Suárez called "known-objects" a few paragraphs ago) those objects do not add an accidental difference to the object of metaphysics either, since they are *completely* distinguished from it.

The particulars of Suárez's opinion aside, it is clear that he disagrees with Eustachius on the two factors—containment of objects and derivation of principles—in relation to which the subalternation of the sciences to metaphysics was determined. This difference of opinion is important. It goes to show that *whether or not* the thinker in question endorsed the subalternation of the sciences to metaphysics, the topic was crucial to a thorough discussion of metaphysics. In other words, endorsed or rejected, *the claim that metaphysics subalternates the other speculative sciences was common.* That claim, however, was based on a concept—subalternation—whose home was in analyzing the mathematical sciences! Renaissance thinkers, following their medieval predecessors, took this concept and expanded its scope from the domain of the mixed-mathematical sciences to the domain of all sciences. Nevertheless, they still took its most exemplary instantiation to be in the mathematical sciences. It was, if you will, a 'mathematical' concept. The expansion of the scope of subalternation and its prevalence in the Renaissance is the central point of this chapter. Looking forward, my analysis of Descartes in Chapter 5 will depend on the prevalence of this expanded use.

4.0 TEMPERED-HYLOMORPHISM AND GALILEO'S SCIENCE OF MATTER

In the second part of this dissertation, I will attempt to apply some of the lessons learned in the first part to the analysis of the new sciences of the seventeenth-century. The structure of this second part mirror the structure of the first part. In the first chapter—on Galileo—I primarily explore the concept of tempered-hylomorphism, as it may shed light on the structure of Galileo's *Discorsi*. In the second chapter—on Descartes—I explore the notion that the sciences are subalternated to metaphysics, as it may shed light on the first two parts of the *Principles*. In both cases, my argument is mainly philosophical. I do not claim to offer insight into authorial intent (although I do offer some circumstantial evidence), but I do claim to offer conceptual tools that aid in understanding certain textual problems in Galileo and Descartes. At any rate, this chapter, apart from providing an application of "tempered-hylomorphism", will provide a working example of a demonstration in a mixed-mathematical, subalternate science.

4.1 INTRODUCTION

Although Galileo's *Discorsi e Dimostrazioni Matematiche, intorno a due nuove scienze* (1638b) is often heralded as his greatest work, the dialogue's first day was largely neglected in Galileo's time and continues to be so in the historical literature.[1] One cause for this neglect is surely the day's convoluted structure. In contrast to the clear-cut propositional style

[1]This chapter, more than others, is indebted to Peter Machamer. I have no doubt that what seems to me obvious about Galileo is obvious because Peter made it so.

of the remaining days, the first day appears as a rambling foray into the host of conceptual issues related to the problem of the cohesion of bodies, often only tangentially. What's more, the lack of an explicit statement as to the purpose of the day's numerous discussions suggests that perhaps there was no unifying purpose, only a mélange of related concerns.[2]

This chapter argues that the first day goes to establish that the study of matter—in particular, the study of the cohesion of bodies—is essentially mathematical. The first day serves as a prelude to the science of the second day by establishing that certain mathematical properties belong to matter *necessarily*.[3] In the second day, Galileo derives a set of theorems concerning matter's resistance to fracture that rest on fundamental assumptions concerning the mathematical structure of matter. The role of the first day is to make these mathematical assumptions plausible. It has long been recognized, of course, that the first day also makes plausible a *physical mechanism* that causally accounts for matter's resistance to fracture. I would like to suggest, however, that what is more important about the first day is that it establishes that this mechanism can be described mathematically.[4] In particular, by showing that matter's physical properties are such that its force of cohesion is finite and acts continuously and uniformly along a given cross-section of a body, the first day justifies the application of the law of the lever—the principle on which the science of the second day is based—to the phenomena of fracture. Once the application of the law of the lever is justified, the causal explanation of matter's resistance to fracture becomes irrelevant. In other words, the second day only requires that certain mathematical properties are true of matter, regardless of their underlying physical explanation. It is for this reason that Galileo can frame his causal account of cohesion as a mere conjecture (*fantasia*) (Galilei 1638b: p. 27 [66]).[5] His attitude towards it echoes his attitude in the third day towards the (lacking)

[2]In contemporary scholarship, the first day is usually discussed in the context of Galileo's matter theory, particularly his so-called mathematical atomism (see, e.g., Smith (1976), Shea (1970)), and in the context of Galileo's role in the development of the infinitesimal calculus (for a concise introduction, see de Gandt (1995), p. 169–176). Although many insights can be gleaned from these and similar studies, they ordinarily focus on particular aspects of the first day and do not attempt to explain the day's contents as a whole or its overall relation to the remainder of the *Discorsi*. Notable exception are Grand (1978) and Palmerino (2001).

[3]I believe the same thesis can be made regarding the relationship of the first day to the third and fourth days, but I will focus my attention on the second day only. Palmerino (2001) studies the relationship between the first and last days.

[4]For Galileo's use of mechanism, see Machamer 1998b,c.

[5]References to the *Discorsi* cite the page number in Drake's (2000) translation, followed by the page number in Favaro's *Opere*, Vol. VIII.

causal account of the heaviness of bodies. In both cases, he is committed to a quantitative description of the phenomena in question while remaining agnostic about its underlying physical explanation. Given the structure of the first two days, I will argue that Galileo's first new science is an instance of mixed mathematics. Under this description, the first day appears not simply as an attempt to mathematize certain properties of matter, but as an attempt to establish that a type of object—matter—possesses physical properties that can be (and by Galileo's light, ought to be) understood mathematically.

Of course, when claiming that the first day establishes that 'certain mathematical properties belong to matter necessarily,' I do not mean to be imputing to Galileo any belief in the *metaphysical* necessity of the properties he studied. Galileo's concern was only with matter as it *actually* is, and questions regarding whether God could have created it differently were foreign to his purposes in the *Discorsi*. I also do not mean to impute to Galileo a belief in "essentialism," the doctrine that "science is concerned with and is able to discover facts about the *inner natures* or *real essences* of things" (Osler 1973: p. 504, emphasis added). As Osler rightly emphasizes, Galileo did not see himself as investigating *inner natures*, but the more-or-less apparent properties of phenomena. I will argue that in the case of matter Galileo considered certain apparent properties to be immutable, to always accompany matter, and to belong to matter simply by virtue of matter being the sort of thing it is.[6] Although "proper attributes" captures some of this meaning and would have been a plausible rendering of Galileo's intention, using a term of art may attribute to Galileo a precision in philosophical matters that is unwarranted. Although he was certainly aware of some of the philosophies of his time, in the *Discorsi* Galileo did not use a technical term to label the properties he studied. Rather, he only characterized them discursively as "eternal and necessary" (Galilei 1638b: p. 13 [51]). I follow his lead and use these terms, but stress again that the more metaphysical implications of "necessary" are nowhere to be found in the *Discorsi*.

This caveat aside, I believe Galileo's statements regarding the properties of matter suggests that the *Discorsi*'s first science ought to be understood as the science of matter—i.e.,

[6]In other contexts Galileo does not focus on similar properties. In the *Letters on Sunspots*, for example, he is happy to examine sunspots by comparing their properties to those of vapors in a pan, with no regard to the properties' necessity.

the science of the properties of bodies *insofar as they are enmattered objects*—not merely the science of the strength of materials.[7]

The distinction is fine but important.[8] As I will argue, by viewing Galileo's science as an instance of mixed mathematics we can see that the subject of that science is a genus—matter. Labeling the science as "the science of matter" makes this fact explicit in a way that "strength of material" simply does not. More importantly, and without reliance on mixed mathematics as an analytical tool, it is clear that Galileo does not present the study of the strength of materials as a free-standing enterprise, but bases it on considerations regarding the nature of matter. In doing so he replaces the received Aristotelian conception of matter with a radically new one—one on which matter itself can have formal, mathematical properties. Thus, although the moniker "strength of materials" accurately represent the contents of Galileo's new science, it fails to capture the significance of his approach to it.

4.2 THE NATURE OF MATTER AND THE NEW SCIENCE

The announced purpose of the *Discorsi*'s first two days is the investigation of the *disproportionate* relation between the absolute size of machines and their ability to function properly, particularly their ability to resist fracture.[9] According to Galileo, the principal difficulty in

[7] See also Machamer 1998c.

[8] The description of the first new science as one concerning the "strength of materials" can be found in, for example, Wisan (1978: 37–38), Segre (1989: 228), and the translation of Crew and de Salvio (Galilei 1638a: 109, 245). This description seems to have originated with the Elzevirs, who in their original table of contents for the *Discorsi* described the first science as one "concerning the resistance of solid bodies to separation" (Galilei 1638b: 9 [47]). The Elzevirs were also responsible for describing the contents of the second day as a "science" both in the table of contents and the title of the entire work, but with some justification: in the *Discorsi* Galileo himself referred to the contents of the second day as a science (Galilei 1638b: 15 [54], 143 [186]).

[9] A wide range of size-sensitive phenomena were well known since antiquity, and attempts to explain them can be found in the pseudo-Aristotelian *Mechanica* as well as the tracts of several of Galileo's predecessors. The very first question of the *Mechanica* deals with the size-dependence of machines by asking why larger balances are more accurate than smaller ones. Commentaries on the *Mechanica* thus tackle the problem of size-sensitivity in this context; see, e.g., Niccolò Tartaglia's *Quesiti et inventioni diverse* (1546) and Giovanni Battista Benedetti's *Diversarum speculationum mathematicarum et physicarum liber* (1585), translated in Drake and Drabkin (1969: pp. 106ff., 180ff.). The *Discorsi* states that larger clocks are more accurate than smaller ones before tacking the problem (Galilei 1638b: 12 [50]). As Renn and Valleriani 2001 convincingly argue, Galileo was inspired to confront the problem of size-sensitivity because of his involvement with Venetian ship-building efforts.

investigating this relation is that Euclidian proportion theory (Galileo's main demonstrative tool in the investigation of mechanical problems) is indifferent to absolute magnitudes and concerns only *ratios*.[10] Galileo puts this difficulty in the mouth of Sagredo, who holds that since machines are at root geometrical, and since in geometry only proportions play an essential role, machines themselves *cannot* be sensitive to absolute size. At this point in the dialogue, the conversation concerns the breaking of scaffoldings:

> [W]hat we [are discussing].... is something commonly said and believed, despite which I hold it to be completely idle, as are many things that come from the lips of persons of little learning... [This is that] one cannot reason from the small to the large, because many mechanical devices succeed on a small scale that cannot exist in great size. Now, all reasonings about mechanics have their foundations in geometry, in which I do not see that largeness and smallness make large circles, triangles, cylinders, cones or any other figures [or] solids subject to properties [*passioni*] different from those of small ones; hence if the large scaffolding is built with every member proportional to its counterpart in the smaller one, and if the smaller is sound and stable under the use of which it is designed, I fail to see why the larger should not also be proof against adverse and destructive shocks that it may encounter.(Galilei 1638b: 11–12 [49–50])

Although with characteristic rhetorical zeal Galileo initially presents this opinion as natural and self-evident, the first and second days of the *Discorsi* go to show that Sagredo is wrong; that is, that machines are sensitive to size and that this sensitivity can be demonstrated geometrically. The possibility of such a demonstration was of the utmost importance to Galileo, given his own methodological commitment to necessarily true, not merely probable, conclusions. If his solution to the problem of the size-sensitivity of machines could not be demonstrated with geometrical necessity, then, by his own lights, it would not amount to a *science*. It is for this reason that Galileo rejected any non-geometrical explanation of size-sensitivity. For example, he rejected the opinions of:

> [S]ome persons of good understanding when, to explain the occurrence in large machines of effects not in agreement with pure and abstract geometrical demonstration, they assign the cause of this to the imperfection of matter, which is subject to many variations and defects. Here I do not know whether I can declare, without risking reproach for arrogance, that even recourse to imperfections of matter, capable of contaminating the purest mathematical demonstrations, still does not suffice to excuse the misbehavior of machines in the concrete as compared with their abstract ideal counterparts. Nevertheless I do say just that... (Galilei 1638b: 12 [50–51])

[10] The very definition of "proportion" was under scrutiny by Galileo and his contemporaries, but in ways that do not impact on this problem; see Palmieri (2001) and references therein.

Galileo was not attacking imagined opponents here. Niccolò Tartaglia, for example, a translator into the vernacular of Euclid and Archimedes and a mechanician in his own right, offered the view in his *Quesiti et inventioni diverse* (1546). In his comment on the first question of the pseudo-Aristotelian *Mechanica*, Tartaglia explained the size-sensitivity of balances in terms of virtual displacements. He held that a larger balance is more sensitive than a smaller one because a given weight generates a greater motion in a larger balance. Because Tartaglia did not consider the time of motion (or, more accurately, he treated motion in differently sized balances as taking place in the same time), he equated a balance's displacement from the equilibrium position with the strength of its motion. Since the extremities of a larger balance cover a greater distance than the extremities of a smaller one when the two go out of equilibrium, Tartaglia concluded that a larger balance is more sensitive. However, he also noted that a consideration of the phenomena "according to reason, all matter being abstracted—as... Euclid was accustomed to do" often contradicts "[a] test [of] that statement materially and with physical arguments... by the sense of sight and with a material balance" (Drake and Drabkin 1969: 106). Tartaglia believed there was an obvious reason for this:

> [T]he cause of this contradiction stems simply from matter; for things constructed or fabricated thereof can never be made as perfectly as they can be imagined apart from matter, which sometimes may cause in them effects quite contrary to reason. (Drake and Drabkin 1969: 106)

That is, Tartaglia believed that embodiment in matter can invalidate geometrical reasoning. Clearly at work in this passage is a conception of matter as that which resists formal, mathematical treatment. Tartaglia makes no allusions to the philosophical underpinning of the conception, but it was a basic tenet of the larger framework of scholastic and renaissance hylomorphism.[11] This framework is crucial for understanding Galileo's work.

[11] Although Tartaglia himself was not educated at a university and made sparse contact with the philosophical tradition of his time, a conception of matter similar to the one he invokes can be traced through the philosophical tradition back to the works of Aristotle (e.g., *Physics* II.8, 199A11, *De Generatione Animalium* IV.3, 767b13ff, 769b10ff. and IV.4). See also Meli (1992) for the differing attitudes of Tartaglia and his predecessors to the Aristotelian tradition. It is interesting to note that Tartaglia believed that the mismatch between mathematical arguments and real machines can be minimized by building machines that are as uniform as possible, but he did not believe the mismatch can be entirely eliminated (Drake and Drabkin 1969: 108–109).

Galileo certainly rejected any appeal to matter akin to Tartaglia's in the explanation of the size-sensitivity of machines. But this is not to say that Galileo thought the matter of machines was unrelated to their size-sensitivity. Rather, it is to say that his response to Sagredo—and with it his solution to the seeming mismatch between abstract arguments and concrete machines—involved a rejection of the very conception of matter with which it was intertwined. By rejecting this conception, Galileo was also rejecting the very idea that embodiment in matter invalidates geometrical reasoning. He held that matter *as such*—not the accidental variations that account for its resistance to formal treatment—was responsible for the size-sensitivity of machines. In the first day of the *Discorsi*, Galileo hypothesized that matter's tendency to cohere and finite capacity to resist breaking forces are responsible for fracture phenomena. Moreover, he held that these can be successfully submitted to mathematical analysis. Offering a full-blown anti-scholastic, single-element theory of matter—as he had done years earlier in the *De Motu*—Galileo wrote:[12]

> I affirm that abstracting all imperfections of matter, and assuming it to be quite perfect and inalterable and free from all accidental change, still *the mere fact that it is material* makes the larger framework, fabricated from the same material and in the same proportions as the smaller, correspond in every way to it except in strength and resistance against violent shocks; and the larger the structure is, the weaker in proportion it will be. And since I am assuming matter to be inalterable—that is, always the same—it is evident that for this [condition] as for any other *eternal and necessary property*, purely mathematical demonstrations can be produced that are no less rigorous than any others. (Galilei 1638b: 12–13 [51], emphasis added)

In other words, in order to treat fracture mathematically, Galileo substitutes the conception of matter inspired by hylomorphism with a conception more amenable to finding the cause of fracture in the formal, geometrical character of matter. In this light, we can see that Sagredo's worry about the limits of abstract geometrical reasoning is a worry about the very constitution of matter. Galileo addresses it head-on by rejecting the conception of matter that leads to the dichotomy between abstract and concrete machines. He rejects the idea that abstraction from the imperfections of matter amounts to "[an] abstraction from *all* matter" (to use Tartaglia's words). Rather, he holds that such an abstraction allows us to arrive at those properties of matter that are "always the same" and belong to matter by "the

[12]For the development of Galileo's conception of matter see Machamer 2005.

mere fact" that it is matter.[13] These properties can be treated mathematically because, like purely mathematical properties, they are inalterable, eternal, and necessary. This fact will become important for my discussion of "tempered-hylomorphism below".

Interestingly, although Galileo's statements regarding the limits of geometrical reasoning are given pride of place at the beginning *Discorsi*, they do not occupy much space in the work. There are few arguments, like the one above, appealing to broad philosophical and methodological principles in the style of *Il Saggiatore* and they are passed over quickly. Rather, Galileo spends most of the first day establishing that the particular properties of matter responsible for the force of cohesion are amenable to treatment by a specific mathematical device, namely the law of the lever. This is perhaps the hallmark of the Galilean approach to the mathematization of nature. Unlike his close contemporaries Descartes and Hobbes, who attempted to justify the mathematical treatment of nature through broad philosophical systems, Galileo's approach was piecemeal, involving the solution to particular problems in particular contexts.

However, and this is the main point of the present section, Galileo's solution to the particular problem of cohesion was nevertheless based on a general conception of matter. That is, it was based on the idea that matter itself possesses a certain structure and this structure (to which I'll return in §4.4) has "eternal and necessary" properties that can submit to mathematical analysis. Galileo's science was profoundly new for precisely this reason: it was a science that not only treated physical bodies mathematically, it treated their very physicality, their matter, mathematically. To put matters more suggestively, if less accurately, Galileo's first science was truly revolutionary because it provided a *formal* treatment of the *material causes* of fracture phenomena. This would have been an outright impossibility on the Aristotelian conception of matter.

It is in this dual treatment that we see signs of the "tempered-hylomorphism" introduced in Chapter 2. In that chapter, we introduced the concept through the largely theoretical considerations contained in the variety of Aristotle's writings. In this chapter, we have a chance to see "tempered-hylomorphism" *at work*. Of course, the appellative would have been anathema to Galileo, but it does shed light on the character of Galileo's efforts, and it is this

[13]For a similar problem and approach in relation to Galileo's study of motion, see Koertge (1977).

light I'd like to focus on. In Galileo's treatment of matter, he jointly treats mathematical and material consideration. Much like an optician, mathematics licenses him to make certain inferences about a material domain, but that domain is not the visual ray, it is matter itself. Also like an optician, the fact that Galileo is reasoning about a material domain makes no difference to the mathematical inferences used: a formal treatment of the subject at hand is licensed through the application of mathematics, but once that application is licensed, the nature of the material domain is irrelevant. As in the case of optics, once a visual ray is described mathematically, no inference are drawn regarding the behavior of a visual ray from that fact that it is visual, only from the fact that is it a mathematical kind. Similarly, once his formal treatment of matter is licensed, Galileo drawn no inferences from the physical mechanism that licenses his treatment, only from the formal tools at hand.

In the next two sections I will show how this formal treatment is based on the law of the lever, how the law comes to gain its foundational status, and how its application to cohesion requires that matter's force of cohesion exemplify particular mathematical properties, namely continuity, uniformity, and finitude. These are the properties of matter which Galileo believes are immutable and belong to matter necessarily.

4.3 THE MATHEMATICAL FOUNDATIONS OF THE NEW SCIENCE

That Galileo aimed to base his new science on secure foundational principles is clear. Early in the *Discorsi* he notes his dissatisfaction with the lack of clear foundations in mechanical treatises and promises to correct their failure. In the mouth of Salviati, he writes:

> I cannot refuse to be of service, provided that memory serves me in bringing back what I once leaned from our Academician [Galileo] who made many speculations about this subject, all geometrically demonstrated, according to his custom, in such a way that not without reason this could be called a new science. For though some of the conclusions have been noted by others, and first of all by Aristotle, those are not prettiest; and what is more important, they were not proved by necessary demonstration from their *primary and unquestionable foundations*. Since ... I want to prove these to you demonstratively, and not just persuade you of them by probably arguments, I assume that you have that knowledge of the basic mechanical conclusions that have been treated by others up to the present which will be necessary for our purpose. First of all, we must consider what effect is at work in the breaking of a stick, or of some other solid whose parts are firmly attached together;

> *for this is the primary concept* [la prima nozione], *and it contains the first simple principle* [il primo e semplice principi] *that must be assumed as known.* (Galilei 1638b: 15–16 [54], emphasis added)

We can learn three things from this passage. First, that the fundamental principle of the new science concerns "what effect is at work in the breaking of a stick." Second, that this fundamental principle is well known and, in some sense, taken for granted. Third, that this principle relates to features of mechanics that are also supposed to be well known. Although the relation between the "first simple principle" and mechanics is not explicated here, other passages suggest that that the principle is a *mechanical* principle. For instance, in the opening to the second day, Galileo writes that:

> In such speculations I take as a *known principle one which is demonstrated in mechanics* about the properties of the rod which we call the lever: that in using a lever, the force is to the resistance in the inverse ratio of the distances from the fulcrum to the force and to the resistance. (Galilei 1638b: 151 [152], emphasis added)

Since this is the *only* non-purely-geometrical principle Galileo assumes at the beginning of the second day, it must be the very principle referred to in the above quotation.[14] The ubiquity of the law of the lever in mechanical treatises also explains why Galileo claimed it was a "known" principle related to known mechanical facts. However, questions still remain: first, what is the "primary concept" exemplified "in the breaking of a stick"? and, second, what does the law of the lever have to do with this primary concept?

Consider the "primary concept" first. After stating that it is necessary to consider what happens when a piece of solid is broken, Galileo describes a solid column from which a weight is suspended:

> To clarify this, let us draw the cylinder or prism AB, of wood or other solid and coherent material, fastened above at A, and hanging plumb; at the other end, B, let the weight C be attached. It is manifest that whatever may be the tenacity and the mutual coherence of the parts of this solid, provided only that that is not infinite[ly strong], it can be overcome by the force of the pulling weight C, of which the heaviness [*gravità*] can be increased as much as we please and that this solid will finally break, just like a rope. (See Figure 1) (Galilei 1638b: 16 [54-55], emphasis added)

[14] In truth, Galileo does not assume the law of the lever, but derives it by considering the behavior of weights suspended from a balance in equilibrium. I will return to this shortly.

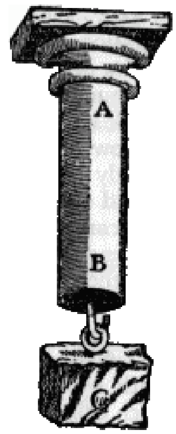

Figure 1: *Longitudinal Pull Case*

Since after this passage the topic of discussion turns to the cause of the cohesion of ropes, it seems that the "fundamental concept" Galileo has in mind is simply that for any given solid, there is a weight which will break the solid when applied longitudinally; or in other words, that for all intents and purposes the "tenacity and mutual coherence of the parts of this solid" has a finite strength that can be overcome by a sufficiently strong counteracting force. Galileo later dubs this the "absolute resistance" of a body to fracture.[15]

Galileo returns to this "primary concept" in Proposition I of the second day. Here, he tries to analyze the resistance of a cantilever in light of what he takes to be the "absolute resistance" evident in the case of a column. Galileo's method for doing so relies on turning the column case—wherein a weight is applied to a solid *longitudinally*—into 'half' of a balance problem—wherein a weight is applied *transversely* to one arm of a balance. The weight acting transversely on the other arm of the balance—the other 'half' of the balance problem—is supplied by the weight of the cantilever. The shift is important not only because it signals yet another instance of Galileo's trademark reliance on the balance,[16] but because it explains the connection between the law of the lever and Galileo's new science. The law of

[15]It seems Galileo conceives of the case of the column as the "absolute" case because in it the force responsible for fracture is applied in the same direction as the resulting motion (Galilei 1638b: 115 [156-157]). See also Footnote 21.

[16]See Machamer (1998b) for a general discussion of Galileo's extensive use of the balance as a model for physical problems.

the lever is the principle used to establish quantitative relationships in the balance problem, and thus, by analogy, is the principle used to establish quantitative relationships between the longitudinal case and the cantilever case. Since Galileo explains the size sensitivity of machines by a series of propositions derived from the cantilever case, the law of lever is the key for solving the problem framed at the outset of the first day. Moreover, since the law is a geometrical principle that licenses the demonstrative inferences Galileo seeks, it has good claim for being an adequate foundation for the new science.

However, in order to establish the foundational status of the law of the lever, Galileo must justify the shift from the longitudinal and cantilever cases to the balance case—the linchpin of the first two days. The shift occurs in Proposition I of the second day. I quote it here in full:

> Let us imagine the solid prism $ABCD$ fixed into a wall at the part AB; and at the other end is understood to be the force of the weight E (assuming always that the wall is vertical and the prism or cylinder is fixed into the wall at right angles) [Figure 2]. It is evident that if it must break, it will break at the place B, where the niche in the wall serves as support, BC being the arm of the lever on which the force is applied. The thickness BA of the solid is the other arm of this lever, wherein resides the resistance, which consists of the attachment that must exist between the part of the solid outside the wall and the part that is inside. Now, by what has been said above, the moment of the force applied at C has, to the moment of the resistance which exists in the thickness of the prism (that is, in the attachment of the base BA with its contiguous part), the same ratio that the length CB has to one-half of BA [Figure 3]. Hence the absolute resistance to fracture in the prism BD, (being that which it makes against being pulled [apart] lengthwise, for then the motion of the mover is equal to that of the moved) has, to the resistance against breakage by means of the lever BC, the same ratio as that of the length BC to one-half of AB, in the prism ... And let this be our first proposition. (Galilei 1638b: 114–115 [156-157])[17]

The proposition states that an analogy can be constructed between the cantilever $ABCD$ of Figure 2 and the balance ABC of Figure 3, which I have constructed from other Galilean drawings to fit Galileo's description, but which is not in the *Discorsi*.

[17]Galileo first introduces this scenario at the outset of the first day, but in a strictly qualitative manner. Not surprisingly, he does so immediately after framing the problem of the applicability of geometry to fracture problems. See (Galilei 1638b: 13 [52]). In modern notation, the proposition states that $f(C)/f(AB) = 2CB/AB$, where $f(C)$ is the force applied to the cantilever at point C and $f(AB)$ is the force of resistance at AB. Since $f(AB)$ is modeled as a weight hanging from a balance, I label it in the diagrams as $W(AB)$. The proposition's last line further states that $W(AB)$ is equal to the force of a weight that can overcome the absolute resistance of the column $ABCD$. I follow Galileo's treatment by suppresses the depth of the beam.

Although Galileo does not spell out the structure of the analogy in any detail, it is clear that he equates what happens "in the breaking of a stick" with what happens when a balance goes out of equilibrium. Intuitively, a piece of solid breaks only when the force of cohesion of its internal parts is overcome by an external pull.

Analogously, a balance goes out of equilibrium only when the force on one of its sides is overcome by a force on its other side. Consequently, if the internal force of cohesion is modeled as the force on one side of a balance and the external pull is modeled as the force on the other side, a piece of solid will break only when the (model) balance tilts to the 'external pull side', i.e., when it goes out of equilibrium in the appropriate direction. This is precisely the chain of reasoning behind Galileo's (already quoted) statement that:

> It is evident that if it must break, it will break at the place B, where the niche in the wall serves as support, BC being the arm of the lever on which the force is applied. The thickness BA of the solid is the other arm of this lever, wherein resides the resistance, which consists of the attachment that must exist between the part of the solid outside the wall an the part that is inside. (Galilei 1638b: 114–115 [156])

The problem with this analogy is that it should to be unclear where to place the weight hanging from the AB arm of the model balance. To see the problem, consider the actual case (Figure 2) and the model case (Figure 3). Since in the actual case weight E is *literally* hanging from the endpoint of one side of a cantilever (Figure 2), it is rather obvious to 'model' it as a weight hanging from the endpoint of one arm of a balance (Figure 3). However, since the resistance at AB acts along the *continuous finite line* AB, it is not immediately clear that it can be represented as a weight, $W(AB)$, hanging *at a point* of the other balance arm, let alone the *mid-point* of that balance arm.[18] To see this, consider as an intermediary another figure I have constructed from Galilean drawings, but that is not supplied in the *Discorsi*, Figure 4.

In Figure 4, the force of resistance at AB (Figure 2) is represented as a weight $W(AB)$ that is hanging from *every point* of one arm of a balance. This is the proper analogue of

[18] I am assuming with Galileo that the resistance of the cantilever can be treated as if localized in AB. In both the longitudinal and the cantilever cases, Galileo believes that the resistance to fracture can be treated if acting along a single cross-section of the body. This assumption is false. See Truesdell (Truesdell: pp. 200–203) and Timoshenko (1953: p. 12).

Figure 2. Is it reducible to the mid-point case (Figure 3)? It turns out that it is, but only under some strict assumptions. These assumptions are implicit in Galileo's proof of the law of the lever.

Galileo's proof of the law of the lever need not concern us in its entirety here. It begins by asking the reader to imagine a beam of material AB hanging by its end-points from a balance in equilibrium HI; i.e., hanging by threads at HA and IB (Figure 5). Clearly,

Galileo writes, if the beam were to be cut at *any* point (say, D) and a thread ED attached from the balance to that point, "[t]here is no question that since there has been no change of place on the part of the prism with respect to the balance HI, it will remain in the previous state of equilibrium" (Galilei 1638b: 111 [153]). Even while ignoring the remainder of the proof, it is clear that the only way to guarantee that D can be chosen arbitrarily is to assume that the weight of the beam is continuously distributed along the segment AB; that is, that the beam has no gaps.[19] Moreover, the only way to guarantee that the beam will maintain its original configuration after the cut, given that D may be chosen arbitrarily, is to assume that the weight is distributed uniformly. With these assumptions—continuity and uniformity—and the law of the lever, however, it is easy to show that $W(AB)$ in Figure 4 can be reduced to $W(AB)$ in Figure 3. Although Galileo does not make these assumptions explicit, it seems they must support his (already quoted) claim that:

> [T]he moment of the force applied at C has, to the moment of the resistance which exists in the thickness of the prism (that is, in the attachment of the base BA with its contiguous part), the same ratio that the length CB has to one-half of BA. (Galilei 1638b: 115 [156])

What is most important about these assumptions (as well as the whole of the balance analogy) is that they make clear that Galileo treats the force of resistance at AB as a *weight*, uniformly and continuously distributed through a beam.[20]

But the proposition does not end here. The aim of the proposition, recall, is to relate the cantilever case to the case of "absolute resistance;" i.e., the longitudinal case. Once again,

[19] I will discuss Galileo's conception of the continuum in the following section.

[20] In truth, the stress distribution along the cantilever section AB is not uniform at the time of fracture (Timoshenko (1953: p. 12)). However, I will assume with Galileo that it is and that the analogy with weight stands.

Galileo does not make any intermediary reasoning explicit, but merely notes after the quote above that:

> Hence the absolute resistance to fracture in the prism *BD*, (being that which it makes against being pulled [apart] lengthwise, ...) has, to the resistance against breakage by means of the lever *BC*, the same ratio as that of the length *BC* to one-half of *AB*, in the prism ... (Galilei 1638b: 114–115 [156-157])

However, the move to absolute resistance requires another crucial assumption; namely, that the force of resistance at AB to a longitudinal pull along $ABCD$ is equivalent to the force of resistance at AB to a transverse pull across $ABCD$. Although this is false, it explains how the "primary notion" enters into Galileo's new science.[21] Galileo believes that all fracture cases can ultimately be related to the longitudinal fracture of a column. The "primary concept" in the longitudinal case was that for any given solid, there is a finite weight that can cause it to fracture. Since all cases of fracture can be understood in terms of the longitudinal case, in all cases the force of resistance can be overcome by a sufficiently large, finite weight. That is, the force of resistance to fracture is itself always finite.

In this way, Proposition I is based on the law of the lever and establishes the relationship between the case of absolute resistance and the case of the cantilever. What the proposition assumes, however, is that the assumptions made regarding the figures above are physically plausible; that is, that the force of cohesion is finite and can be unproblematically described as acting uniformly and continuously along line segment AB of Figure 2, just as a finite weight an be described as being uniformly and continuously distributed along the hanging beam AB in Figure 5.

[21] Although it is an open question why Galileo endorses this assumption to begin with, I can offer the following hypothesis: Galileo conceives of fracture as the motion of one part of a body away from another. In the cantilever case, he believes fracture occurs along cross-section AB. Consequently, he believes that the force responsible for fracture is equal to the force required to move the portion of the cantilever to the right of AB away from the part embedded in the wall. In the longitudinal case, he also believes fracture occurs along a single cross-section. Consequently, he believes that the force responsible for fracture is equal the force required to move the portion of the column below the breaking point away from the portion of the column above that point. If the prism and cantilever are constructed to have appropriately similar dimensions, the resulting motions will only be different in orientation. It stands to reason that the forces responsible for them would also be identical.

4.4 THE APPLICATION OF MATHEMATICS JUSTIFIED

We come now to the question with which this chapter began: what is the role of the first day of the *Discorsi*? I claim that the first day shows that in all formal, geometrical respects, matter's force of resistance to fracture is like the force of a uniformly distributed, continuous, and finite weight hanging from a balance. In other words, the first day establishes that the mathematical properties necessary for the balance analogy to be applied are true of the force of cohesion. Galileo's physical explanation of the force of resistance is important to keep in mind here. Galileo hypothesizes that this force is caused by indivisibly small vacua that are interspersed among the indivisibly small particles of matter. Because of the *horror vacui*, these vacua pull together the particles of matter; that is, they resist the particles' separation. In essence, Galileo explains the cohesion of matter through nature's abhorrence of a vacuum and his peculiar conception of the structure of matter. However, his task is to show that this structure of vacua and particles has the aforementioned mathematical properties. Although I cannot examine the first day in detail here, a sampling of its contents and a brief description of their relation to the day's larger purpose are sufficient to make my case.

First, resistance to fracture must be finite. This is not only because reasoning concerning an infinite quantity would be impossible by Galileo's lights, but because in order to model resistance on weight, it should be possible to correlate any given measure of resistance to a measure of weight. Among others, Galileo's 'hanging bucket' experiment is meant to show that such a correlation is possible (Galilei 1638b: 23, [62]). In this experiment, Galileo asks the reader to imagine a piston inserted into a cylindrical cavity that has been evacuated of air and turned such that its open end is pointing downwards. Galileo holds that the piston will be held in place by the force of the vacuum within the cylinder and that this force could be overcome by hanging a bucket on the piston and placing within it a sufficiently large weight. Since in this experiment resistance is not simply *correlated* to weight, but is *directly measured* by it, the experiment allows for a straightforward substitution of a weight-magnitude for a resistance-magnitude—precisely as done in Proposition I. Also in support of the claim that the force of resistance is finite is Galileo's well-known discussion of Aristotle's wheel—the centerpiece of the first day (Galilei 1638b: 28–58 [68–97]).[22] Galileo describes by means of Aristotle's wheel how an infinite number of indivisibles can sum to a finite quantity. The first day's remaining discussions on the nature of infinity, the continuum, and indivisibles go to support Galileo's treatment of Aristotle's wheel and bolster his argument that, despite his unique explanation of cohesion by an infinity of indivisible vacua, on a macroscopic level that force can be treated as finite and measurable.

Second, the force of resistance must act continuously and uniformly through any given cross-section of a body. Continuity and uniformity are perhaps the most important properties of resistance, since without them Galileo would not be able to make the analogy between the force of resistance along line segment AB (Figure 2) and a weight hanging from a balance arm (Figure 4). Continuity and uniformity are established primarily through Galileo's discussion of Aristotle's wheel. There, Galileo shows that his physical explanation of cohesion is such that regardless of the density of a material, the distribution of indivisible vacua and material particles will always be its uniform. Thus, the force of resistance to fracture will always be uniform. Moreover, Galileo provides an extended discussion of the nature of the continuum meant to show that the continuum is comprised of an infinite set of indivisible full and empty points (Galilei 1638b: 28–58 [68–97]). Of course, Galileo's theory of the continuum is subtle and merits an extended discussion which I cannot provide here. Suffice it to say that the theory is clearly meant to support Galileo's account of cohesion and to illustrate that it is mathematically tractable.[23]

The remaining portions of the first day go to make the physical properties of cohesion plausible in themselves and immune to objections from the study of motion. Particularly, since Galileo argues that cohesion is due to the *horror vacui*, he must argue that a vacuum is physically possible (Galilei 1638b: 26–28, [66-67]). It is to this end that he cites the example of two blocks of marble that slide easily across one another but can only be separated by a great force. The example provides experimental verification that a vacuum can be found in nature (Galilei 1638b: 19–20, [59]), as does the aforementioned 'hanging-bucket' experiment.

[22]For Galileo's treatment of Aristotle's wheel see Drabkin (1950), Wallace (1989), and Palmerino (2001).

[23]Galileo addresses several issues of great importance in the medieval literature on infinity and infinitesimals; namely, the paradoxes of unequal infinities, the existence of first and last instants, the nature of the verbs "to begin" and "to cease," and the problem of maxima and minima. See Kretzmann (1982).

It is also to this end that Galileo undertakes the long discussion of bodies falling in resistive media—a discussion which takes up more than half of the first day. The discussion is necessary since the impossibility of motion in a vacuum was traditionally taken as evidence against its existence. In order to show that a vacuum *can* exist, Galileo must therefore frame an anti-Aristotelian theory of motion.[24]

Although extremely sketchy, I take this brief roster to suggest that there exists a connection between the discussions of the first day and the application of the law of the lever in the first proposition of the second day. Given this connection, however, more could be said about the seemingly disparate character of the discussions, particularly the fact that some seem purely mathematical (e.g., the nature of the continuum), while others seem to involve a good deal of physical reasoning (e.g., the explanation of condensation and rarefaction). Clarifying the nature of these problems will lead us back to one of this chapter's original concerns and the more precise reason Galileo's first new science is best understood as the science of *matter*. Here, we can finally tie matters back to the notion of tempered-hylomorphism introduced in Chapter 2.

4.5 TEMPERED-HYLOMORPHISM AND SUBALTERNATION

As the standard formulation goes, the subalternate, middle, or mixed-mathematical sciences are sciences in which physical objects are considered *qua* mathematical. However, as explicated in Chapter 2, this formulation misses an important element of the *modus cognoscendi* of the mixed-mathematical sciences; namely, that they not only consider physical objects as mathematical, but they somehow treat physical (or, more narrowly, material) consideration jointly with mathematical ones. My contention here is that Galileo's first new science exemplifies this *modus cognoscendi*. I do not contend, however, that Galileo saw it as such,

[24]The structure of this portion of the first day can be found in Galileo's manuscripts as early as 1590, Drake and Drabkin (1969). The first day's discussions on the nature of motion in a vacuum can also be understood as showing that *weight* is a necessary property of matter, and that it can be treated mathematically. I have not stressed this in this chapter since my focus is on those features of the first day that are necessary for the second day, not for the third and forth days. Palmerino (2001) focuses on the relevance of elements of the first day to Galileo's science of local motion.

even though I have been contending that his purpose in the first day was to establish those mathematical properties of matter required for the application of the law of the lever in the second day.[25] My aim is only to show that the reasoning of the first two days of the *Discorsi* does in fact fit the structure of arguments in the mixed sciences.

In order to see this, let's repeat some of our previous comments regarding the mixed-mathematical sciences. Recall that the goal of a science in the Aristotelian tradition was to supply demonstrably true claims regarding its subject-matter by showing what properties belong to that subject-matter by virtue of the type of thing it necessarily is, not by virtue of any accidental features that may be true of it (See Chapter 2 and the discussion of Suárez in Chapter 4). As Aristotle wrote in the *Posterior Analytics* I.9:

> We understand a thing nonincidentally when we know it in virtue of that according to which it belongs, from the principles of that thing as that thing. For example, we understand something's having angles equal to two right angles when we know that to which it belongs in virtue of itself, from that thing's principle. (translated in Lennox (1986: 40, [76a4-8]))

Taking the triangle example, Aristotle holds that we understand a particular isosceles (or any other instance of) triangle by coming to understand it as belonging to the class of triangles *in general* (i.e., the genus "triangle"); and by the principles of triangularity (i.e., geometrical principles) we come to understand that *any* triangle's angles are equal to two right angles. We thus come to know that an *isosceles* triangle's (or any other triangle's angles) are equal to two right angles. Even without explicitly putting the inference in syllogistic form, it is plain that the middle term here is 'triangle': it serves to connect a particular isosceles triangle, through the geometrical features of triangularity, with the features of all triangles. In the case of the subalternate sciences, however, the middle term takes on a dual character. The quote above continues:

[25] Although Galileo has been portrayed as a Platonist (e.g., Koyré 1939), a positivist (e.g., Mach 1960: 151-191), and everything in between, (see Wallace (1992) and Feldhay (1998) for overviews), in the past twenty years a leading interpretation based mostly on the work of William Wallace has characterized him as following an Archimedean inspired version of the philosophy of science outlined by Aristotle in the *Posterior Analytics* and elaborated by the Jesuits of Galileo's time—the same philosophy of science that gave rise to the notion of the "subalternate sciences." Of course, there is no overall consensus regarding Wallace's interpretation, and disagreements still abound about whether Galileo's method constituted proper apodictic *scientia*, whether it followed the medieval *regressus* more than Aristotle's original proclamations, whether it constituted Galileo's *juvenilia* more than his mature thought, etc.; see McMullin (1978, 1983), Jardine (1976), Wallace (1976) and Pitt (1978). For the purposes of this discussion, I put aside these larger historical questions and take for granted that some light can be shed on Galileo's work from the perspective of the tradition of the subalternate sciences. See also the second caveat made in the introduction to this chapter.

> Hence if that too [the thing's principle] belongs in virtue of itself to what it belongs to, the middle term must be in the same kind. If this isn't the case it will be as the harmonical properties are known through arithmetic. In one sense such properties are demonstrated in the same way, in another sense differently; for that it is the case is the subject of one science (for the subject-kind is different), while the reason why it is so is of a higher science, of which the per se properties are the subject. (translated in Lennox (1986: 40, [76a8-13]))

Aristotle outlines two cases: one in which the thing studied and the principles used to study it are of "the same kind" and one in which they are not, as in the subalternate science of harmonics. In the latter case, which is certainly familiar by now, the thing studied is natural, and the principles used to study it are mathematical. The middle-term in these cases "picks out the description of the natural object in virtue of which it has a certain mathematical property; that property is a per se property of a natural kind qua being a mathematical kind" (Lennox 1986: 41). In other words, in a subalternate science natural objects must be described in a way that attributes to them via their (natural, physical) genus mathematical properties which they have simply in virtue of being described mathematically.

We can see that the first two days of the *Discorsi* fit the mold of a subalternate science. In the first day, a natural kind—matter—is described in a way that attributes to it mathematical properties that belong to it by its very structure, simply because of the kind of thing it is. As argued above, the uniformity, continuity and finitude of matter's force of cohesion are such properties. The overall task of the *Discorsi*'s first two days is to argue that bodies fracture because they are en-mattered, that because they are enmattered they possess certain physical properties that can be described mathematically, and that these mathematical properties entail additional mathematical properties of fracture. The first of these is accomplished by Galileo's rejection of the Aristotelian conception of matter and his framing of a new conception on which fracture occurs even when matter is "free from all accidental change" and because of "the mere fact that it is material" (Galilei 1638b: 12 [51]). The second of these is accomplished throughout the first day, as Galileo establishes that his conception of matter entails that matter can be described mathematically and treated geometrically. Galileo's *physical* arguments in support of his new conception of matter show that the mathematical properties of matter are in fact properties of a natural, physical kind that is "always the same" (Galilei 1638b: 12 [51]). It is for this reason that the first day seems to oscillate between purely mathematical considerations (e.g., regarding infinity and indivisibles) and purely physical ones (e.g., regarding condensation and rarefaction). The third of these is accomplished by the second day. In the first proposition of the second day, the law of the lever is applied to the phenomena of fracture given the mathematical properties of matter justified by the first day. The remainder of the second day draws out the implications of the first proposition and establishes the theory of fracture.

But where is "tempered-hylomorphism" in all of this? Haven't I just argued that once a formal, mathematical description of matter is licensed, no more material considerations regarding matter are necessary? Wouldn't this collapse Galileo's new science into geometry? In fact, it doesn't. There are two reasons for this, a cheap and not so cheap one.

The cheap reason the new science does not collapse into pure geometry is because the demonstrations of the new science never do elide the fact that they are about a certain *natural* kind—they are, after all, about matter's force of cohesion. That force is treated mathematically, but it is quite obvious in all demonstrations of the first new science that there is real, actual force of cohesion in under consideration. In other words, Galileo's new science is parallel to optics in the following sense: In optics, geometrical reasoning applies to visual rays because when considered as a mathematical kind, the natural kind "visual ray" has a certain mathematical structure. Such reasoning constitutes *optical* reasoning. If, however, it is forgotten that optical reasoning is applicable *because* the kind "visual ray" has a certain mathematical structure (or more simply, because light rays have a certain mathematical structure), then optical reasoning becomes nothing more than geometrical reasoning. But this is never the case in optical reasoning, since non-mathematical features of visual rays are always used to interpret geometrical reasoning, that is, to explain how geometrical reasoning ultimately applies to light and vision.[26] The same is true for Galileo's science. Although his demonstrations are geometrical, their material subject matter is never forgotten. Of course, one may object to this and point out that, according to my own argument, Galileo 'turns' the force of cohesion into the force applied to a lever, and so, by my own lights, it is not something that is under explicit consideration in Galileo's demonstrations. This is where

[26] McKirahan 1978 calls those principles that allow for the interpretation of geometrical results in visual terms "determinations".

the not-so-cheap reason comes in. The no-so-cheap reason is that, apart from the force of cohesion, the natural kind that is at the center of the first new science possess another real, irreducible property that is involved in all of the science's demonstrations: weight. In other words, Galileo's demonstrations in the new science depend not only on the continuity and finitude of matter's force of cohesion, they depend on the fact that matter is essentially drawn towards the center of the earth. There is no demonstration in the new science that can make sense without recognition of this fact, and the very foundation of the science—the law of the lever—crucially depends on it. In this way, Galileo's new science is even a better example of the tempered-hylomorphism of the mixed-mathematical sciences than optics had been. This is because with Galileo's new science it is quite impossible to ever 'forget' the natural kind to which a mathematical treatment is applied. A physical property of that natural kind is involved essentially in the very foundations of the science.

A further clue to the character of Galileo's first new science as a kind of mixed-mathematical science, one that is not related to tempered-hylomorphism, is supplied by the fact that the question with which the first day begins—why large machines are different than small ones—is solved only in Propositions VI and VII of the second day (Galilei 1638b: 12, 120–124 [50, 163-166]). This structure echoes the familiar Aristotelian distinction between the *fact* and the *reasoned fact*, reviewed a few pages ago. In the first day, only the *fact itself* is mentioned, and it is noted as a known mechanical fact. The explanation of the fact—"the reason why it is so"—is delayed until the second day. In other words, although Galileo began the first day with the promise that "it can be demonstrated geometrically that the larger [machines and structures] are always proportionately less resistant than the small" (Galilei 1638b: 13 [51]), it is only at the closing of Proposition VI of the second day that he admits the desired demonstration had been reached:

> *Simp.* This proposition strikes me as not only new but surprising... I should have thought it certain that their moments [i.e., the moments of large and smaller cylinders and prisms] against their own resistances would be in the same ratio. [as their sizes].
> *Sagr.* This demonstrates the proposition which, as I said at the *beginning of our discussions*, seemed then to reveal itself to me through shadows. (Galilei 1638b: 122, [164], emphasis added)

The proof is not available until the second day because Galileo had first to establish the relevant mathematical properties of matter on which the mathematical demonstrations of the second day rest. In this sense, the convoluted discussions of the first day enable the transition between the *mere* fact of fracture and the *reasoned* fact concerning it. It is crucial to note that only the second day, as a series of propositions, properly constitutes Galileo's new *science*, but its formulation depends on the deliberations of the first day. The expository structure of this new science fits well with the ideas that "that it is the case is the subject of one science..., while the reason why it is so is of a higher science".[27] In this case, Galileo's higher science is used to explain facts about fracture that are known from mechanical practice. Put in terms of "subalternation", the higher science is a science that, as Eustachius and Suárez put it, is "partially subalternated" to geometry—one that is based both on purely geometrical principles and on the law of the lever. The subject of this subalternate science are the "eternal and necessary" properties of matter. Since this science is framed in response to worries about the applicability of geometrical reasoning to matter (one again, in the language of subalternation, since this science is framed in response to worries about the possibility of 'borrowing' principles from geometry and 'applying' them in the domain of matter) and since this science is concerned with objects insofar as they are enmattered, it should be understood as the science of matter *as such*.

In the next chapter, I will consider another new scientist, Descartes, and how the set of concepts used to analyze the mixed-mathematical sciences may shed light on his work. Whereas this chapter was a mirror to chapter 2 and its focus on tempered-hylomorphism, the following chapters will be mirror to chapter 3 and the idea that the relation of metaphysics to the remainder of the speculative sciences could be explicated in terms of "subalternation".

[27]This distinction between a *propter quid* and *quia* demonstration, originating in the *Posterior Analytics*, was a staple of Renaissance thinking on logic and method. See Randall 1940, Gilbert 1963, Schmitt 1969, Jardine 1976, Wallace 1995, Barker and Goldstein 1998, 2001, Poppi 2004, South 2005. The translation of Aristotle's original is taken from Lennox 1986: 40. Lennox also treats the topic of demonstrations *qua* and *quid* in relation to Galileo's discussion of projectile motion.

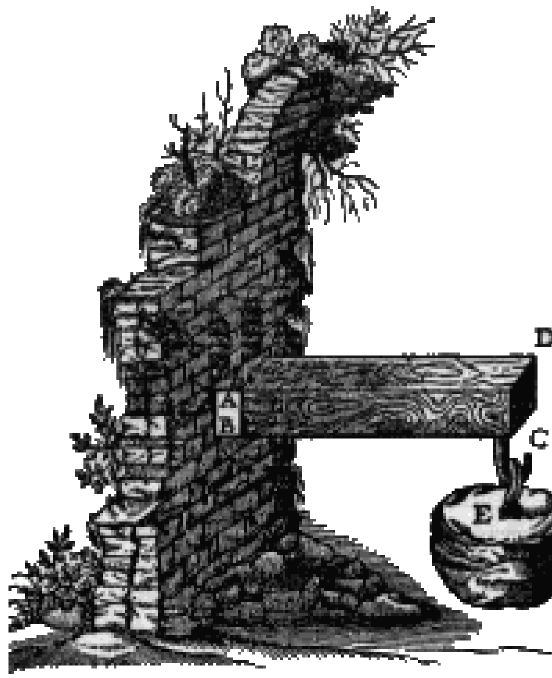

Figure 2: *Cantilever Case*

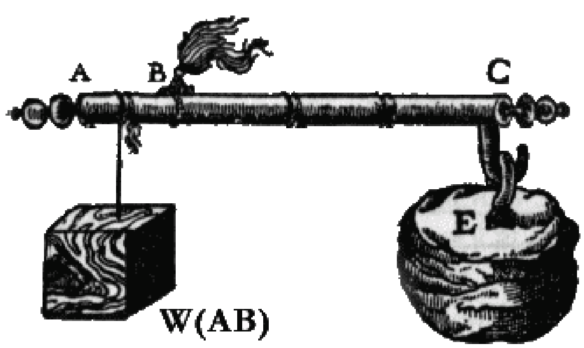

Figure 3: *The Resistance to fracture in the cantilever case modeled as a weight applied transversely at the mid-point of balance arm AB.*

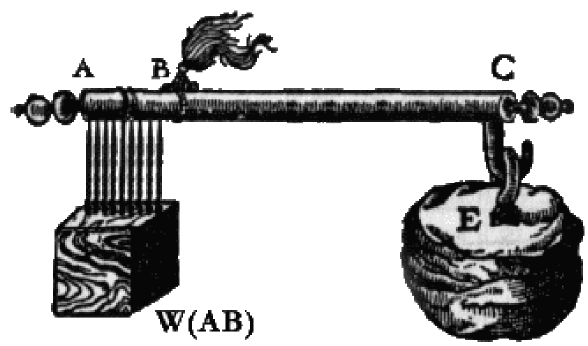

Figure 4: *Resistance to fracture in the cantilever case modeled as a weight applied transversely and continuously along balance arm AB.*

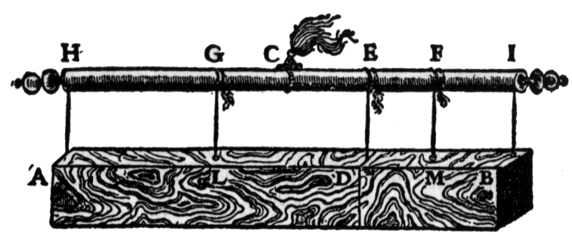

Figure 5: *A continuously and uniformly distributed weight hanging from a balance.*

5.0 THE SUBALTERNATION OF PHYSICS TO METAPHYSICS IN DESCARTES

In the previous chapter, I argued that the notion of tempered-hylomorphism makes sense of Galileo's endeavor in the first two days of the *Discorsi*. Apart from demonstrating the utility of the concept, I wanted to offer in that chapter a working example of a mixed-mathematical demonstration, one that allows for the application of the principles of one domain to a different one. In the present chapter, I will argue that concept of subalternation (originally taken from reflection on the nature of mixed-mathematical demonstration) may make sense of the structure of Descartes' scientific system vis-à-vis metaphysics and physics, as it is presented in the *Principles*.

5.1 INTRODUCTION: A DEMARCATION PROBLEM?

In the preface to the French Edition of the *Principles of Philosophy* (1647) Descartes famously proclaimed:

> [T]he whole of philosophy is like a tree. The roots are metaphysics, the trunk is physics, and the branches emerging from the trunk are all the other sciences... (AT IXB 14; CSM I 186)

As hackneyed as it now is, the image put in vivid terms the foundationalist project Descartes had set himself since his philosophical youth: to base natural science, and indeed, all human knowledge, on indubitable grounds. In his earliest (unpublished) philosophical work, the *Regulae ad directionem ingenii* (c. 1619–1628), Descartes presented the quest for certainty in terms of intuition and deduction. In his mature years, he substituted clear and distinct ideas for intuitions as the guarantors of certainty, but remained focused on the deduction of complex claims from more fundamental ones. In this part of his life, Descartes articulated a set of metaphysical theses in the *Meditations* (1640) and reworked them as the foundation of a complete scientific system in the *Principles* (1644, 1647). Although Cartesian scholarship has long ago abandoned the idea that Descartes intended to deduce the *entirety* of his scientific system from metaphysical principles,[1] it is still commonly thought that he intended to deduce the *principles of physics* directly from metaphysics.[2] Descartes promoted this view throughout his life. For example, in an oft-quoted letter to Mersenne (27 May 1638) he writes that a "demonstration of the principles of physics by metaphysics" is "something I hope to do some day but which has not yet been done" (AT II 141–142, CSMK 103). Although Descartes merely announces his intention in this letter, two years later he seems confident that he had accomplished his task. In a letter most likely to Gibieuf (11 November 1640), he writes that "the little book on metaphysics which I sent you [i.e., the *Meditations*] contains all the principles of my physics" (AT III 233, CSMK 103). And in the *Principles* itself, he writes that the physical theses defended in the work "have been deduced [*deducta sint*] in an unbroken chain [*continuâ serie*] from the first and simplest principles of human knowledge", i.e., from the metaphysical principles articulated in the first part of that work (AT VIIIA 328, CSM 290).[3]

But there's a snag. If we accept this view of Descartes' system we face a curious demarcation problem. In Chapter 2 and 3, we saw that in the scholastic-Aristotelian tradition there existed a clear demarcation line between *metaphysica* and *physica*. In that tradition, the principles of metaphysics and physics were not mutually deducible because they concerned essentially different types of *objects*: the first concerned essentially changeless abstract objects, while the second concerned essentially changing, enmattered objects. For Descartes,

[1] Most contemporary scholars believe that parts III and IV of the *Principles* were justified by appeal to experience and various theoretical virtues (e.g., wide explanatory scope, unification, etc.). See Clarke 1982, Hatfield 1985.

[2] For orthodox presentations of this view, see Schouls 1972, Buchdahl 1969, and, in a way, Garber 1978. For a notable exception, see Williams 1978. The view is also implicit in several contemporary interpretations of Descartes which we will review shortly.

[3] See also AT I 144, CSMK 22; AT I 563, CSMK 87; AT I 476, CSMK 77, AT III 233, CSMK 157 and the remainder of the preface to the French *Principles*, AT IXB 2–12, CSM I 181–190. I will defend the claim that the first part of the *Principles* provides *metaphysical* principles later in this chapter.

however, the principles of physics are deducible from those of metaphysics in a *continuâ serie*. Consequently, there seems to be a *prima facie* problem which the contrast with Aristotelianism brings into view (or, more to the point, a problem an Aristotelian reader would have been sensitive too): if Descartes' physical principles are deduced from metaphysical principles, how are they differentiated from those metaphysical principles? Regardless of what view one takes concerning the individuation of the sciences (Aristotelian or otherwise), what makes Descartes' physical principles particularly *physical*, given that they are deduced from premises of an inherently *metaphysical* nature? Put in yet another way, if Descartes' physics somehow 'grows' from his metaphysics—as the tree image suggests—how are the two disciplines defined and where is the line between them drawn? On what grounds? With appeal to what tenets of Cartesianism? Needless to say, there are ready-made demarcation criteria like "physics is about material things, metaphysics is not", but my point is simply that such seemingly obvious standards ought to be evaluated in light of Descartes' claims that his metaphysics and physics constitute a continuous whole. Most certainly, we must ascertain what might be meant by "deduction," "metaphysics", and "physics" in the Cartesian context.

This problem will be my focus in this chapter. In order to resolve it, I will a new interpretation of Descartes' deduction of the laws of nature, the centerpiece of his deduction of physics from metaphysics. Although this is well-traversed territory, the interpretation is novel insofar as it relates Descartes' arguments to a model of deduction that was well-known in the seventeenth-century but has since faded from view.[4] The model is derived from the scholastic concept of "subalternation"—a concept used to describe the interrelations of individual sciences within the scholastic hierarchy of sciences—and is more akin to what we nowadays mean by "application" (as in *applied* mathematics, *applied* ethics, etc.) than it is to contemporary notions of entailment or implication. As I argued in previous chapters, Descartes' readers would have been aware of this model of deduction and would have even expected to find it in a work of comprehensive scope such as the *Principles*. Consequently, I believe we ought to explore whether there are possible advantages to interpreting the *Principles* in this way. To preview, the thesis I will ultimately defend is that if we understand

[4]See Chapter 1 for a review of the literature.

Descartes' claims to have "deduced" his scientific system according to the model of subalternation, we can solve the demarcation problem in a way that does justice to the text, as well as come to a new understanding of the overall structure of Descartes' scientific system. In particular, we can come to view Descartes' physics as an "applied metaphysics".[5]

In this chapter, I'll focus on better describing the demarcation problem and the meaning of "deduction" (§5.2), offering preliminary definitions of "physics" and "metaphysics" as they are found in the *Principles* (§5.3), and articulating the way in which Descartes' concepts of body and motion are or are not 'deduced' from his metaphysics (§5.4). At the end of §5.4 I will reconsider the problem of demarcation in light of these finding. In §5.5 , I will analyze the overall structure of Descartes' arguments for his laws of nature, and argue that they ought to be analyzed on the model of subalternation.

Two caveats must be made before we begin. First, although I appeal to evidence from Descartes' letters and unpublished writings, my aim is not to establish authorial intent. As stated in Chapter 1, my interest is in the way Descartes' contemporaries would have read his work, particularly in the range of interpretations that would have been open to them that perhaps have since been closed to us. Consequently, to the extent that it is possible, I try to analyze the *Principles* not as a window into Descartes' developing thought, but as a free-standing text. I thus weight textual evidence from the *Principles* and other published sources more heavily, and use unpublished evidence merely to shed light on what I believe can be plausibly extracted from the *Principles* itself. That said, in the conclusion to this chapter, I offer circumstantial evidence to suggest that my interpretation would not have been entirely alien to Descartes.

Second, although I sometimes use the terms "physics", "principles of physics", and "the laws of nature" interchangeably in order to ease exposition, they have well-defined extensions. "Physics" refers to the whole of Descartes' natural science (including the rules of collision, his theories regarding the vortices, the magnet, the functioning of the human body, etc.). The "principles of physics" refers to those elements of Descartes' physics that are the conceptual and deductive foundations of the remainder of his natural science. These include his concepts

[5] Although Kant used the locution "applied metaphysics" (*angewandter Metaphysik*) to refer to certain portions of physics, I do not mean to suggest a continuity with his view. See *Metaphysical Foundations of Natural Science*, Chapter 1 (Phoronomy), Explication 1; and Friedman: Ch. 1 (*manuscript*).

of body, motion, and the laws of nature. "The laws of nature" are simply one sort of "principles". Where a distinction between terms makes a difference, I make it.

Caveats aside, in order to attack the demarcation problem, I should first make explicit the assumptions on which it does and doesn't rest. Although thus far I have only described an "orthodox" interpretation of Descartes' deductive system, there are in fact numerous interpretations which differ in detail and emphasis. Clarifying the assumptions on which the demarcation problem rests will serve to clarify the range of interpretations to which it applies. As will become clear in the following section, certain interpretations are quite vulnerable to it, others less, but the problem cannot be dismissed entirely. In particular, I want to make clear that the demarcation problem is *not* an artifact of an overly stringent view of Cartesian deduction (although in order to solve it I will try to be as stringent as possible in interpreting "deduction ", at least for a little while). Because of this, I spend much of the next section arguing against what I call the *logical containment* assumption. The section will close off by considering more lenient views.

5.2 THE ASSUMPTIONS BEHIND THE DEMARCATION PROBLEM

To begin with, the demarcation problem relies on the assumption that there must be something in the *content* of the principles of a given domain that make them the principles of *that domain*. For example, it seems that there must be something in the content of Descartes' metaphysical principles that makes them pertain to metaphysics, something in the content of his physical principles that makes them pertain to physics. I'll call this the *content dependence* assumption. By "content" I mean what Descartes signals by the "objective reality" of an idea. In the *Second Replies*, for example, Descartes defines the objective reality of an idea as "the being of the thing (entitatem rei) which is represented by an idea, in so far as this exists in the idea" (CSM II 113, AT VII 161). Although this is a rather oblique formulation, in the *Fifth Objection* Gassendi helps us understand it. There, Gassendi notes that an idea of some thing has objective reality iff that idea "contains representatively nothing which is not in fact in the thing itself", and, more properly, that it contains "merely. . . that part of the thing of which the intellect has acquired knowledge" (AT VII 285, CSM II 199). In short, the objective reality of an idea relies on the content of that idea. Explicitly, the objective reality of an idea is not *identical* with its content, but is a function of it: the greater the representational content of an idea, the greater is its objective reality.[6] This makes an important difference, since some ideas may have representational contents that do not correspond to a positive reality, as, for example, in sensory ideas. As Descartes explains to Burman, when ideas represent something that does not have the reality the idea represent it to have, it is materially false (Descartes 1976: 11).[7] Nevertheless, the content of the idea relates directly to its purposed objective reality and Descartes extensive use of the concept of objective reality indicates that we are committing no historiographical sin by discussing the purported "content" of Cartesian ideas.

In these terms, the content-dependence assumption states that the content of Descartes' principles is what makes them either 'physical' or 'metaphysical'. This assumption is independent of whatever theory of meaning we ascribe to Descartes. Apart from what has already been said regarding the nature of "principles" in chapters 2 and 3, there are two reasons to accept this assumption. First, it seems that the other main candidate for making a set of principles the principles of a given domain, namely, their form, is ruled out by the fact that none of Descartes' principles seem to conform to a particular formal structure or a set of formal features. Second, a pragmatic or arbitrary coupling of a set of principles with a particular domain is ruled out by historical evidence. Throughout his published and unpublished writings, Descartes used "metaphysics" and "physics" in a consistent fashion, suggesting that the disciplines and their principles are defined by their essential characteristics, not by virtue of external relations. One may argue, of course, that for Descartes the relevant external relations are neither pragmatic nor arbitrary, but nevertheless external. One may argue, for example, that metaphysical principles are metaphysical not by virtue of their content, but by virtue of their relation to human thinkers; say, by virtue of the *process* by which they are acquired or their *function* in our thought. I'll return to this option later: as Descartes changed the very meaning and function of "metaphysics", this option

[6] Despite these distinctions, in the secondary literature 'objective reality' is sometimes used to refer directly to an idea's content. See, e.g., Ariew et al. 2003: 71.

[7] See also *Meditation III*, CSM II 24ff., AT VII 32ff.

cannot be ruled out initially, but, I will later show, neither does it destroy the validity of the content-dependence assumption.

At any rate, the content-dependence assumption is important because it allows us to frame the demarcation problem more precisely. Given the content-dependence assumption, we can say: Supposedly, the principles of Descartes' metaphysics and physics have some content, the content we grasp when have acquired knowledge of them. This content determines the status of these principles as either 'metaphysical' or 'physical'. Thus, we can conveniently describe the content of Descartes' metaphysical principles as 'metaphysical content' and the content of his physical principles as 'physical content'. However, one set of principles is also supposed to be deducible from the other. So a question arises: how can we understand Descartes' claim that from the 'metaphysical content' of the principles of metaphysics he has 'deduced' the 'physical content' of the principles of his physics? How are the two sorts of content, and the disciplines they found, demarcated from one another? We can also think of this problem in language developed in Chapter 3. There we saw that sciences were often distinguished according to the objects they treat. The content of a science—what we grasp when we grasp the truths of that science—would thus be suited to the type of objects with which it was concerned. Another way of putting the problem is thus: how can truths about the objects—or "intellectual objects", as Suárez put it—of physics derive from truths about the objects of metaphysics?

The content-dependence assumption is also important because is allows us to formulate a strategy for addressing what I've called the demarcation problem. If the content of Descartes' principles determines their status as metaphysical or physical, what we must do is 'follow the content': we must see precisely how the content of Descartes' physical principles is related to (deduced from, grows out of, etc.) the content of his metaphysical principles. This is precisely what I will do in sections 5.4 and 5.5. In other words, an easy solution to our problem would be to show that in the *Principles* physical content *does in fact* emerge from metaphysical principles. I intend to try to execute this project (someone disingenuously, I must admit) in order to see how and where it falls short.

A second assumption, an assumption on which this problem does *not* rest, is that the Cartesian deduction of physics from metaphysics is non-ampliative; or, as Descartes would have put it, that its physical conclusions are somehow 'contained' in its metaphysical premises.[8] I'll call this the *logical containment* assumption.[9] For much of the present section, I'll argue against this assumption in order to make clear that my formulation of the demarcation problem is not an artifact of an overly stringent (and ahistorical) view of Cartesian deduction.[10]

The *logical containment* assumption is most likely to be accepted by those who understand Descartes' deduction of physics from metaphysics on a certain model of logical inference (perhaps that of the demonstrative syllogism) or synthetic geometrical demonstration. I'll address these in turn. To begin with, on a fairly standard early-modern understanding of the relationship between premises and conclusions in a logical inference (particularly a demonstrative syllogism), the conclusions of such an inference are 'contained' in its premises.[11] If the deduction of physics from metaphysics is understood in *this* way, then the content of Descartes' physical conclusions must be somehow 'contained in' his metaphysical premises. If this were the case—which I believe it is not—the demarcation problem would be exacerbated and become a true dilemma. In order to meaningfully demarcate physics from metaphysics given the containment assumption, we would need to either reject the idea that the deduction is actually a *deduction* or reject the idea that it proceeds *from metaphysics*. That is, we would need to either reject the idea that the content of the deduction's conclusions is logically contained in its premises or accept the idea that the deduction contains non-metaphysical elements that can provide the principles of physics with non-metaphysical content. For one who endorses the logical containment assumption and respects Descartes' statements regarding the relationship of his physics to metaphysics, *prima facie*, either option seems

[8] In general, we need not think that the claim that a certain inference is non-ampliative entails the claim that the conclusion of that inference is 'contained' in its premises or that its conclusion is somehow 'implicit' is its premises. However, as far as I can see, this is how Descartes thought about non-ampliative inferences. I'll address the historical grounds of this assumptions momentarily.

[9] This assumption is different than the *causal* containment thesis found in common notion IV of *Second Replies* (AT VII 165, CSM II 116; see also AT VII 41, CSM II 28–29). Where as the causal containment thesis concern the objective reality of ideas, the logical containment thesis concerns the content of which the objective reality of ideas is a function.

[10] The logical containment assumption should be distinguished from the *causal containment* assumption that can be found in the *Meditations*. There, Descartes holds that a cause must formally or eminently contain its effect, since the reality of the cause must be greater than or equal to the reality of the effect. See *Meditation* III and Normore 1986.

[11] This was a conception of syllogism held, for example, by Spinoza (*Letters* 4 and 60) and more famously by Hume (*A Treatise of Human Nature* 3.1.1.27.), but not by Aristotle.

unpalatable.

Steven Nadler (1990) seems to hold such a view of Descartes' deduction of the laws of nature. He holds that the content of Descartes' physical laws are (a) deduced from metaphysical premises alone and that (b) the deduction itself is non-ampliative. He characterizes the deduction of the laws of nature as follows: "Let $\mathrm{G}_1 \ldots \mathrm{G}_n$ represent certain proposition regarding God, including his existence, his essential attributes, and his role as the primary cause of motion in the universe; and let $\mathrm{M}_1 \ldots \mathrm{M}_n$ represent certain metaphysical propositions regarding the nature of matter and of motion. Then from $\mathrm{G}_1 \ldots \mathrm{G}_n$ together with $\mathrm{M}_1 \ldots \mathrm{M}_n$, we can infer [the laws of nature]" (Nadler 1990: 362). He also notes that "God's essence [and 'certain metaphysical propositions regarding the nature of matter and of motion'] uniquely and a priori determines what the *content* of the laws of nature must be, prescribing one particular set of laws to the exclusion of all others" (Nadler 1990: 363, original emphasis). Since propositions regarding God are unquestionably *metaphysical* propositions, and since Nadler unequivocally states that proposition regarding matter and motion are also metaphysical, he clearly holds that all premises of the deduction of the laws of nature are metaphysical. That is, he accepts (a).

Although Nadler does not explicitly address (b), his adherence to the logical containment assumption is presupposed by the overall strategy of his essay. In his 1990, Nadler argues against two types of interpretation of the deduction of the laws of nature (in the *Principles* only). According to the first, the laws of nature are logically and rigorously entailed by metaphysics in general, and by God's essence in particular. Nadler interprets this to mean that the *content* of the laws, as well as their *truth*, are derived from metaphysics. Nadler views this as the orthodox position in Cartesian scholarship and attributes it to Guéroult 1953 and Garber 1978 (who has since disavowed the position). According to the second type of interpretation against which Nadler argues, the inference from metaphysics only secures some general features of the laws of nature—i.e., that they are conservation laws—but does not determine the content of the laws. According to this view, determining the content of the laws requires further appeal to experience and other non-metaphysical factors. Williams 1978 champions this view. Nadler finds problems with both positions and carves out an intermediate view. He holds that the derivation of the *content* of the laws and the confirmation of their *truth* should be treated independently, and argues that Descartes intended to *deduce the content of the laws of nature from metaphysics alone*, but left the confirmation of their truth to experience and other non-metaphysical factors (Nadler 1990: esp. 378ff.).

What's important to us about this strategy is that it makes clear that, according to Nadler, the metaphysical premises of the deduction of the laws fully determine the content deduction's conclusions. Without this claim, his position would collapse into William's view, a view he wishes to reject. This assumption, however, leads directly to (b) and the characterization of Descartes' derivation of the laws of physics from metaphysics that results in the demarcation dilemma.

Nevertheless, there is reason to believe that the deduction of physics from metaphysics should not be understood in this way. We can see this by considering Descartes' statements regarding the demonstrative syllogism. Although Descartes is clear that he intends to "deduce" physics in general and the laws of nature in particular from metaphysics, his contempt for the syllogism is (and was) well known. Echoing Ramist criticisms, he repeatedly disparages the utility of the syllogism and maintains that it is an inappropriate tool for acquiring new knowledge, that is, for invention or discovery. He does so precisely because he believes that the conclusions of a deduction are 'contained' in its premises, and so a syllogism—and, supposedly, any inference that suffers the same flaw—cannot yield truly novel knowledge. In the Preface to the French edition of the *Principles*, for example, he urges students of philosophy to study logic, but not "the logic of the Schools", since it only "teaches ways of expounding to others what one already knows..." (AT IXB 13, CSM I 186). Similarly, in Rule 10 of the *Regulae*, written about twenty years earlier, he states:

> [The] art of reasoning contributes nothing whatever to knowledge of the truth... [D]ialecticians are unable to formulate a syllogism with a true conclusion unless they are already in possession of the matter [i.e., content][12] of the conclusion, i.e., unless they have previous knowledge of the very truth deduced in the syllogism. (AT X 406, CSM 36–27)

Despite Descartes' clear life-long contempt,[13] however, one could nevertheless argue that these quotes do not support my claim. They merely show that Descartes' disparaged the

[12]CSM translate "nisi prius ejusdem *materiam* habuerint" as "unless they are already in possession of the *substance* of the conclusion". This is a clear distortion, and fails to indicate that Descartes' usage is consonant with classical logical hylomorphism.

[13]See also *Discours* (AT VI 17), *Second Replies* (AT VII 140) and *Conversation with Burman* (AT V 147).

syllogism as an instrument of *discovery*, yet the *Principles*—and the deduction of physics therein—was written as an aid to *teaching*. Why couldn't have Descartes constructed his substitute to the scholastic textbook using the tool he conceived to be adequate for "expounding to others what one already knows"?

Stephen Gaukroger (1989) follow precisely this line of reasoning in his study of Descartes' concept of deduction. He writes there:

> Descartes's criticisms of traditional syllogistic, and, by extension, deductive reasoning generally turn on the argument that it cannot yield new truths. [However] [t]he apparently deductive structure of the *Principles*... is not actually in conflict with this since what the *Principles* provide is not a means of producing new truths but rather a systematic natural philosophy into which our physical results, arrived at by hypothetical and experimental means, must be incorporated. (Gaukroger 1989: 116)

However, Gaukroger's position also seems untenable, although less obviously so. Without casting doubt on his characterization of the *Principles* as providing a natural philosophical framework capable of integrating novel results, one need only point to the fact that when Descartes was addressing the subject of syllogistic logic in the Preface to the French edition, he did not bother to point to the relevance of the syllogism for understanding the very work prefaced. Wouldn't the preface have been a fitting occasion to appeal even further to the scholastic audiences Descartes was trying to reach? It seems so, yet when describing how a reader should prepare to read his philosophy, Descartes dismisses the study of syllogism (which "corrupts good sense rather than increasing it") and recommends the study of the type of logic that is best practiced by solving "simple questions like those of mathematics" (AT IXB 13–14, CSM I 186). It seems unlikely, although far from impossible, that Descartes would have attacked the worth of the syllogism so ferociously in a work that had been constructed around it.

But perhaps Descartes had a different model of deduction in mind, a model that is not syllogistic yet still admits of the logical containment assumption. Descartes' recommendation to study the type of logic best practiced by solving "simple question like those of mathematics" might provide an important clue. Perhaps the structure of the *Principles* was not inspired by the syllogism, but by simple geometrical problems—say, like the ones found in the first book of Euclid's *Elements*. As I will show shortly, Descartes' also understood *these* sorts of demonstrations as non-ampliative and according to the containment metaphor. And thus, if he had intended to follow *this* method in the deduction of physics from metaphysics, the demarcation problem would still be a full-blown dilemma, for exactly the same reasons as before.[14]

This reading of 'deduction' in the *Principles* is suggested by several sources. To understand it, we must take a detour through the *Second Objections* to the *Meditations*. In the *Second Objections*, Descartes is asked by the objector—Jean-Baptiste Morin, perhaps—to "set out the entire argument [concerning the existence of God] in a geometrical fashion, starting from a number of definitions, postulates, and axioms" (AT VII 128, CSM II 92).[15] As part of Descartes' complex answer, he distinguishes between two methods of geometrical demonstration, the analytic and the synthetic. He holds that he had used the analytic method in the *Meditations* itself, and interprets the objector's request for an argument in the "geometrical fashion" to mean that a synthetic presentation of the same results is sought. Although Descartes ultimately supplies the requested synthesis in his "Geometrical Appendix", more important for us is his characterization of the synthetic method itself. He writes:

> Synthesis... demonstrates the conclusion clearly and employs a long series of definitions, postulates, axioms, theorems and problems, so that if anyone denies one of the conclusion it can be shown at once that *it is contained* [*contineri*] in what has gone before, and hence the reader, however argumentative or stubborn he may be, is compelled to give his assent. (AT VII 156, CSM II 111, emphasis added)[16]

Certainly, this description of fits what we find in Euclid's *Elements*.[17] Moreover, it seems to suggest that the logical containment assumption must be true of, and perhaps essential to, synthetic demonstrations. After all, the function of a synthetic demonstration—to compel the assent of an argumentative reader—is achieved *by virtue* of the fact that the conclusions

[14] Interestingly, it seems that Beck 1952: 77 assumes that Descartes understood the method of the deductive syllogism and the method of synthetic geometrical demonstrations to be the same. Although this assumption is not endorsed by any contemporary commentator I am aware of, it not without historical merit; see Sasaki 2003: 64ff on Clavius' treatment of Euclid.

[15] See Garber 1995 for the claim that Morin is the author of the request.

[16] See also *Regulae*, Rule 14

[17] The literature on 'analysis' and 'synthesis' in early-modernity is vast. In the subsequent paragraph, I am only scratching the surface, and leave completely undiscussed the relation of synthesis to *a posteriori* and *a priori* reasoning, to methods of teaching and exposition, to the medieval methods of *resolutio*, *compositio*, and *negotio*, and the relation of all these to exemplars in geometry and the natural science. For a review of the literature, see Ch. 1.

of such a demonstration are "contained in what has gone before".[18]

But does this description fit what we find in the *Principles*? Clearly, the *Principles* does not employ postulates, theorems and problems. However, many commentators have relied on a passage from the so-called *Conversation with Burman* which suggests that, nevertheless, the *Principles* follows the synthetic method of demonstration.[19] In the *Conversation*, after having drawn a distinction between the method of teaching (*ordo docendi*) and the method of discovery (*ordo inveniendi*), a distinction similar but not identical to the *Second Replies*' distinction between the synthetic and analytic method, Burman reports that "In the *Principles*, [Descartes] teaches and *makes use of synthesis*" (AT V 152, emphasis added). Although a vast literature exist on the meaning of this phrase and its relation to the *Second Replies*, there are two dominant approaches. The first, pursued by Martial Gueroult, Edwin Curley and J. M. Beyssade, holds that the *Principles* is in fact synthetic, but that the meaning of 'synthesis' is different than what is presented in either the *Conversation* or the *Second Replies*.[20] Consequently, the 'containment' notion central to the *Second Replies* is no longer considered by them to be an essential element of 'synthesis', and the logical containment assumption fails to be true of Descartes' method in the deduction of physics from metaphysics. The second approach, pursued by Daniel Garber, holds that the attribution of a synthetic method to the *Principles* is erroneous, and the attempt to reconcile Descartes' proclamations is historically wrong-headed.[21] According to this line of thought, no notion of 'synthesis' fits

[18] Later on Leibniz and Kant would reverse the relation of syntheticity to containment. Concerning judgments, Kant writes: "Either the predicate B belongs to the subject A, as something which is (covertly) contained in this concept A; or B lies outside the concept A, although it does indeed stand in connection with it. In the one case I entitle the judgment analytic, in the other synthetic" (*Critique of Pure Reason* A 6–7, Kant 1965: 48), see also Leibniz to Arnauld, Leibniz 1973: 62.

[19] Other passages suggest similar readings, but the passage from Burman seems to be particularly pregnant because of its explicit mention of synthesis in relation to the *Principles*. In Part II of the *Discours*, for example, Descartes writes: "Those long chains composed of very simple and easy reasonings, which geometers customarily use to arrive at their most difficult demonstrations, had given me occasion to suppose that all the things which can fall under human knowledge are interconnected in the same way" (AT VI 19, CSM I 120). This and other similar passages suggest a 'geometrical', synthetic reading of Cartesian deduction.

[20] Guéroult 1953, Beyssade 1976, Curley 1977. A murkier instance is John Cottingham, who translates the phrase from the *Conversation* quoted above as "In the *Principles* his purpose is exposition, and his procedure is synthetic" and the "*ordo docendi*" as "the order of exposition" (Descartes 1976: 16). His reason for turning *docere* into 'expose' is to avoid the contradiction that results from Descartes' description of 'synthesis' as the method of teaching (docere) in the *Conversation* and 'analysis' as the method of teaching (docere) in a portion of the *Meditations* not quoted here (Descartes 1976: 67ff,). As with Curley and Guéroult, his purpose is extract from Descartes a coherent account of both analysis and synthesis.

[21] Garber 1982, 1995. This approach also challenges the reliability of the *Conversation* as a Cartesian text.

both the structure of the *Principles* and Descartes' statements (and lack thereof) regarding it, and so it seems plausible that the "the discussion of the *Principles*. . . [should] lack any reference *at all* to the distinction between analysis and synthesis" (Garber 1982: 62, original emphasis). If we accept this line of thought, the logical containment assumption also fails: since the *Principles* does not follow the synthetic method of demonstration, *a fortiori* it does not admit of the logical containment assumption presented as part of that method in the *Second Replies*. Of course, the *Principles* may admit of the logical containment assumption by following *another* model of demonstration, but, as far as I am aware, we have hereby covered and dismissed the two most likely models of Descartes' deduction to admit of this assumption: the syllogistic model and the synthetic, geometrical model.

So where does this leave us? I have gone through a lengthy treatment of the logical containment assumption in order to make clear that it is not a plausible assumption. I did so for two reasons. First, I did so in order to stress that for those who accept the assumption, the demarcation problem is unavoidable and is, in fact, a dilemma. Consequently, the questions I ask in this work (and the answers I provide) are critical for the consistency of their views. Second, I did so in order to make clear that my own understanding of the demarcation problem—which I will return to momentarily—does not result from an overly stringent view of Cartesian deduction, one that depends on accepting the logical containment assumption. It is because I treat a much weaker formulation of the demarcation problem that my results should apply equally well to the stronger dilemma associated with the logical containment assumption.

As a side-effect of this mostly negative investigation, however, what should have also been made clear is that the very nature of "demonstration" or "deduction" in the *Principles* is problematic. In fact, one of the major positions on the subject holds that Descartes' concept of deduction is *irreducibly* wider than our own, and that attempts to narrow it are historically suspect. Charles Larmore, for example, writes:

> It is to be remembered that the Cartesian idea of deduction is broader than the logical concept of deduction—it covers any sequence of propositions where we perceive 'clearly and distinctly' that the conclusion follows from the premises. (Larmore 1980: 9)

Similarly, Daniel Garber writes:

See also Ariew 1987.

> In calling the structure [of Descartes' scientific system] deductive I do not mean to say that it is deductive in precisely the modern sense, or that it is deductive in any precise sense at all...there is no in principle reason why [for Descartes] a deduction cannot be an ampliative inference in the modern sense of the term, as, for example, the *cogito* seems to be. (Garber 1993: 91)[22]

If this is the case, however, what becomes of the demarcation problem? How should we view the interface between Descartes' physics and metaphysics once the logical containment assumption is rejected?

To close this section, I'd like to suggest that although more lenient views of Cartesian deduction save us from the *dilemma* associated with accepting the logical containment assumption, the demarcation problem still exists, albeit in a weakened form. Even with an ampliative view of Cartesian deduction, we are still faced with the challenge of articulating how Descartes' metaphysical theses are amplified into physical ones in the *Principles*. We ought to be able to do that, just as we are able to articulate how the *cogito* amplifies a phenomenological thesis into a thesis about what there is.[23] In particular, given the content dependence assumption, we ought to be able to describe how, and at what points of Descartes' "continuous" argumentative chain, the *content* of Descartes' physical principles emerges from the *content* of his metaphysical principles, as the tree image suggests. Such a description amounts to a solution to the weaker demarcation problem.

Notice that I am not asking here how Descartes' metaphysical principles *guarantee the truth* of his physical principles, nor am I asking for how his metaphysics is used to *classify concepts* used in physics as genuinely veridical, explanatory or specious. Although the truth-vouching and classificatory functions of Cartesian metaphysics are important—and, no doubt, one of Descartes' greatest achievement was to alter the functions of metaphysics—in order to answer the demarcation problem we must undercover how Cartesian metaphysics executes a different function; namely, that of contributing to the content of the theses deduced from it.[24] This function *may not be the dominant function of Cartesian metaphysics*

[22] Although in this work Garber only discusses Descartes' conception of inference in the *Discours* and earlier work, I believe he would happily apply his analysis to the later *Principles*. See also Clarke 1977, 1982.

[23] Of course, considerable controversy surrounds this task; see Markie 1992 for a synopsis of debates.

[24] Clarke 1989 has argued that Cartesian metaphysics has *only* classificatory and truth-guaranteeing functions in grounding Cartesian physics. I will show that this is not the case. Moreover, I believe this is a severely deflationary reading of Descartes' usage of "deduction" in the contents of the relationship between metaphysics and physics. The verb "deduce" is often used by Descartes in a way that can only be interpreted

(and we should not expect it to be, since in the *Principles* Descartes believes a large stock of our idea are available innately and thus their representational content is native to the mind),[25] but we should at least see whether metaphysics fulfills this function in some modest way. This might be an overly stringent view of the claim that physics is 'deduced from metaphysics'—certainly more stringent than the view that a deduction from metaphysics is 'any sequence of propositions with metaphysical premises where we perceive 'clearly and distinctly' that the conclusion follows from the premises'—but I'd like to adopt this stringent view in order to see what are the most minimal changes we can make to it, and still make it true.

As promised in the introduction, I will do so by examining those of Descartes' arguments that constitute the interface between his metaphysics and physics, and determine in what fashion each one contributes to the content of his physics. First, however, the demarcation problem demands that I must better define what falls under the banner of "physics" and "metaphysics" in the *Principles*, and locate the interface between the two.

5.3 PHYSICS AND METAPHYSICS IN THE *PRINCIPLES*

Of the four parts of the *Principles*, only the latter three concern physics. The first of these—"On the Principles of Material Things"— details the *principles* of physics. These include the nature of material body, place, space, motion and rest, and the laws of motion; the very principles Descartes purports to derive from metaphysics. Parts III and IV of the *Principles* draw out the consequences of Part II for "The Visible Universe" and for inanimate and animate bodies on "The Earth", respectively.

This structure and subject matter would have been identified as "physics" by Descartes'

as suggesting that the process of deduction generates the representational content of its conclusions. For example, in a letter that is likely to Vatier (22 February, 1638), Descartes notes that he has "deduced" observational data from the principles of his physics (AT I 563, CSM III87). Certainly, Descartes does not mean to suggest that he has proved certain observation data to be *true*, nor that he has classified certain findings *as* observational data (whatever that would mean), but rather that he has generated the *content* of that data through a deduction. In general, it seems that nearly all of Descartes' statement to have "deduced" previously unknown *effects* from *causes* can be read in this way.

[25] See Machamer and McGuire (*forthcoming*) for an account of Descartes' nativist "epistemic stance", and also Schmaltz 1997 and references therein.

scholastic-Aristotelian contemporaries. As we saw in Chapters 2 and 3, according to the the scholastic-Aristotelian division of theoretical sciences and the theory of abstraction on which it was based, physics was ordinarily defined as the study of enmattered objects *insofar as they are enmattered*. Thus, the fact that the overt subject matter of the last three parts of the *Principles* concerned "material things" would have alerted Descartes' readers to its nature as a "physics". However, as we also saw in Chapters 2, in scholastic metaphysics the notion of material embodiment was intimately tied to the notion of change. Consequently, Descartes' contemporaries would have expected a "physics" to treat not only enmattered objects, but enmattered objects *insofar as they are subject to change*. Notably, these two components of physics are not on equal footing in the *Principles*. The reason is that the metaphysical coupling of matter and change was severed by Descartes, and thus, it is possible to isolate those parts of the *Principles* that concern the principles of material bodies *insofar as they are enmattered* (*Principles*, Part II §1–22) from those parts that concern the principles of material bodies *insofar as they are subject to change* (*Principles*, Part II §23ff.). This makes our current task easier, since we can cleave the question regarding the relation of Descartes' physics to his metaphysics in two neat parts: first, we can ask how the principles of his physics *qua* the study of enmattered objects derive from metaphysics, and second, we can ask how the principles of his physics *qua* the study of change in material objects derive from his metaphysics.

What about the nature of Cartesian metaphysics? The literature here is vast, but the following points are clear.[26] First, Descartes' considered the range topics covered in the *Meditations* to be "metaphysics". The full title of the work—*Meditations on First Philosophy*—intimates as much. However, Descartes was also explicit, and repeatedly. As early as 1630, he writes to Mersenne that he had begun work on "a little treatise on metaphysics" which contains proofs regarding "the existence of God and of our souls when they are separate from the body, from which their immortality follows" (AT I 182, CSMK 29). These are the same subjects he would later address in the *Meditation* (although the immortality of the

[26] Anglo-Saxon scholarship seems less concerned than its continental counterpart with with the status of Cartesian metaphysics and the status of Descartes as a metaphysician. For treatments of both issues, see the recent Kambouchner 2005 and the more classic Marion 1986 and Guéroult 1953.

soul is addressed only briefly in the *Synopsis*, AT VII 12–13, CSM II 9–10).[27] Similarly, in 1639, when he first formulates his plan to distribute copies of his yet unnamed (and likely unwritten) *Meditations* in order to receive objections, he claims that the work will contain "a great part of my *metaphysics*" (AT II 622, CSMK 141, emphasis added). In the *Principles*, he refers to the *Meditations* as his "Metaphysical Meditations" (*in Meditationibus Metaphysicis*), a phrase that was also to become the French title of the work (*Méditations Metaphysiques*) (AT VIII 17, CSM II 203). What is important to note about this is that although the designation "metaphysics" is not used to refer to any part of the *Principles* within the *Principles* itself, the range of subject matters treated in the *Meditations* makes a reappearance in Part I of the *Principles*.

Second, although Descartes' conception of metaphysics was novel in many ways, a good number of the topics treated in the *Meditations* or the first part of the *Principles*—e.g., the existence and attributes of God, the immortality of the soul, the notions of substance and attribute—were not foreign to traditional metaphysics. Although there was no consensus on the scope of metaphysics within scholasticism, the discipline was primarily conceived as the study of either separable substances—like God and the soul—or the study of those features of substances that were most general, like the notion of being, the distinction of substance and accident, necessity, contingency, and truth.[28]

Of course, despite the similarly of topics of scholastic-Aristotelian and Cartesian metaphysics, Part I of the *Principles* goes under a novel title which we cannot ignore. Descartes does not call this part 'On the Principles of Metaphysics', but "On the Principles of Human Knowledge". What should we make of this? As I noted earlier (see footnote 24 and surrounding paragraphs), in the *Meditations* and the *Principles* Descartes pushed the classificatory

[27]The subtitle of the first edition of the *Meditations* is "in which is demonstrated the existence of God and the immortality of the soul". However, Descartes does not actually provide a demonstration, but notes that "the premisses which lead to the conclusion that the soul is immortal depends on an account of the whole of physics", an account which he does not intend to provide in the work (AT VII 13-14, CSM II 10). In the second edition of the *Meditations*, the subtitle only advertises a demonstration of the "*distinction* between the human soul and the body".

[28] In scholasticism, metaphysics was commonly divided into "general" and "special" species. General metaphysics concerned the study of being *qua* being and special metaphysics concerned natural theology, cosmology and rational psychology. Although Descartes seems to forgo general metaphysics almost exclusively to the favor of special metaphysics in the *Meditations*, the *Principles* does contain a good deal on the nature of substance and truth. For a detailed treatment of general metaphysics in the *Principles* see Chappell 1997.

and truth-vouching functions of metaphysics to the fore. In other words, the *dominant*, perhaps definitional, function of metaphysics in those texts is to prepare the ground for the attainment of truth in the sciences by specifying which of our innate concepts can be properly used in those sciences and by determining how truth itself is to be recognized. *This* project surely is concerned with the sources of human knowledge. However, I believe this cannot be the *only* appropriate characterization of Descartes' metaphysics, on pain of making some of Descartes' statements regarding the nature of metaphysics entirely vacuous. Quite often, Descartes describes his metaphysics not by its function vis-à-vis human ideas, but by the range of topics it covers, i.e., by its content. In the previous paragraphs, we saw how some of these statements echoed the scholastic definition of special and general metaphysics in the context of the *Meditations*. Even after Descartes had dubbed the subject of Part I of the *Principles* "human knowledge", Descartes continues to define metaphysics topically, not by its foundational role. In the Preface to the *Principles*, for example, he differentiate between the principles of "immaterial or metaphysical things" and the principles of "corporeal or physical things". Clearly, in this context Descartes equates the metaphysical with the immaterial and the physical with the corporeal (AT IXB 10, CSM I 184). Of course, in the same preface he also clearly states that the "Principles of knowledge... [are] what one can call first Philosophy or Metaphysics". What are we to make of this variance? Instead of selecting a single way to define Cartesian metaphysics, I believe it is more accurate to holds that Descartes defines the scope of his metaphysics both by its foundational role in relation to human knowledge *and* by the range of topics it covers. My concern in this chapter is not with metaphysics' foundational role, but with the range of topics that fall in its purview. Thus, although the title of Part I of the *Principles* is significant, it does not stand in the way of the analysis to be offered.

Having defined the scope of metaphysics and physics according to the range of topics with which they are concerned, we can ask about their connection. In the *Principles*, the deductive link between Descartes' metaphysics and physics centers on, although is not exhausted by, the deduction of the laws of nature. The deductive link centers on these laws because, by Descartes own lights, "all the variation in matter, or all the diversity of its forms, depends on motion" (Principles, §23, AT VIII 52). The laws that govern motion are thus the central explanatory principle of Descartes physics.[29] Tersely put, Descartes believes that God is the cause of motion, and so an analysis of His attributes can lead to the discovery of the way His effects—i.e., worldly motions—occur. Embodied in this terse formulation are three elements that require further attention, each of which relates Descartes' metaphysics to the principles of his physics in different ways. First, there is the claim that God is the cause of motion. According to Descartes, however, God is not the cause of motion unqualifiedly, but the "universal and primary" or "general" cause (*Principles* II §36, AT VIIIA 61, CSM I 240). In addition to God, the causal structure of the world consists in "secondary and particular" causes. These are the "laws of nature" (*leges naturae*), the very product of the derivation with which we are concerned (*Principles* II §37, AT VIIIA 62, CSM I 240). Second, and facilitating the causal gradations above, there is a distinction between motion taken as an effect *independently of its cause*, and motion taken as a *caused* effect. Descartes holds that understanding the way bodies move requires appeal to the *cause* of motion, but understanding the *nature* of motion does not. The nature of motion, in other words, does not depend on the causal structure in which it is embedded. This leads to the third element required for understanding Descartes' derivation of the laws of nature. For Descartes, the *nature* of motion must be "referred to" the nature of material substance because it is a *mode* of material substance, but it need not be "referred" to the *cause* of motion. I will take up these thee issues in turn, but backwards, considering how each is derived from metaphysical considerations.

5.4 BODY AND MOTION

At the core of Descartes' conception of body (or 'corporeal body' or 'matter') is the idea that the nature of body is geometrical extension and that the properties of body are geometrical

[29] Despite being the central explanatory principles, the laws are not by themselves sufficient for determining the actual motions of parts of matter. In order to carry out this latter task, Descartes supplies (in the French edition) seven rules of collision that further specify the way in which the laws of nature (the third law in particular) apply to particular configurations of matter.

properties.[30] Although the relation of this core idea to Descartes' broader metaphysical and epistemological teachings underwent significant changes in the course of his career, the idea had been with him for decades.[31] In Rule 14 of the *Regulae* (written about 1628-29), for example, he writes regarding the sentence 'Body possesses extension':

> Here we understand the term 'extension' to denote something other than 'body'; yet we do not form two distinct ideas in our imagination, one of extension, the other of body, but just the single idea of extended body. So far as the fact of the matter is concerned I might just as well have said 'Body is extended', or better still 'That which is extended is extended'. (AT X 444, CSM I 60)

This identification continues, albeit with varying degrees of clarity and philosophical sophistication, throughout Descartes' life. In *Le Monde*, he presents the equivalence of corporeal body and extension as part of his fabled "new world". Although the fable allows him to frame his beliefs tentatively, the matter-as-extension thesis becomes more explicit and divorced from the cognitive overtones that permeate the *Regulae*. Extension, Descartes write, is "not an accident", but body's "true form and essence" (*Le Monde* §6 24, AT XI 36). In the *Principles*, the view reaches its most articulated form. Extension, Descartes argues there, is the "principal attribute" of body, the "one principal property which constitutes its nature and essence, and to which all its other properties are referred" (*Principles* I §53, AT VIIIA 25, CSM I 210). Although the principal attribute is still only an *attribute*, it is distinguished from the substance to which it is attributed by a distinction of reason only. It is an attribute "without which the substance [to which it is attributed] is unintelligible" (I. §VIIA 30, CSM I 214). Body, on this view, is a *res extensa*, an extended substance.

[30] I say "at the core" because there is disagreement over whether geometrical extension exhausts the nature of matter. See footnote 32.

[31] See Machamer and McGuire 2006.

How is the content of this concept of body derived from metaphysical considerations?[32] In the *Principles*, Descartes provides two arguments for his concept of matter which repeat to some degree arguments found in the *Mediations*. Following Daniel Garber, we'll call these the argument from elimination and the complete concept argument.[33]

5.4.1 THE ARGUMENT FROM ELIMINATION

The argument from elimination, which in the *Principles* is presented under the heading "The nature of body consists not in weight, hardness, colour, or the like, but simply in extension", is a terser formulation of an argument found in the *Second Meditation* (AT VIIIA 42, CSM I 224; AT VII 30, CSM II 20). There, the reader is asked to consider a piece of wax. That piece of wax, like any piece of wax (AT VII 31 CSM II 21), changes its smell, taste, color, shape, sound, and hardness when heated. I stress that the piece of wax is like any piece

[32] Notice that I am not asking how the claim that material body *exists* derives from metaphysical considerations. Although this is certainly part of Descartes' physics, I am concerned with the *representational content* of his concept of matter, not its existential status.

For the sake of completeness, however, I should note that in the argument for the existence of body one does not find the content of Descartes' conception of body derived from metaphysical considerations. In order to prove the *existence* of material things in the *Principles* and *Meditations* (more on the difference between the works shortly), Descartes argues from the following premises. First, we passively seem to have ideas of bodies existing outside us, and so we naturally believe in their existence because we do not believe that we, nor, in the *Meditations*, any other creatures besides us, are responsible for generating such ideas. That is, we assume that the external bodies of which we have ideas are themselves responsible for our ideas of them. Second, we have a clear and distinct idea of what those bodies are like; namely, they are bits of geometrical extension, but bits of geometrical extension that exist as objects in the world. Third, God is non-deceiver. Consequently, since God is a non-deceiver, bodies conceived as bits of reified extension must in fact be the responsible for our perceptions of them, and thus must exist.

Clearly, this chain of reasoning depends on Descartes' conception of God as a non-deceiver, a solidly metaphysical idea. However, the role of God in this reasoning is merely to vouchsafe the formal reality of objects who objective reality as ideas is arrived at, insofar as this argument is concerned, independently of him. Thus, at least within the context of this argument, the representational content of Descartes concept of matter is not derived from metaphysics.

On another note, I intentionally used the language of "responsibility" in presenting this argument in order to equivocate between the causal role attributed to bodies in producing our perceptions of them in the *Meditations* and in the *Principles*. In the *Meditations*, Descartes holds that external bodies have an "active faculty" that gives rise to our ideas of them (AT VII 79, CSM II 55). In the *Principles*, however, there is no reference to faculties of bodies, but only to perceptions that "come to us" from external sources (AT VIIIA 41, CSM I 223). Although the difference here is tied up with one of the most hotly debated issues in *Cartesiana*; namely, the causal status of force in Cartesian physics and its seat (or lack thereof) in bodies, it is incidental to our purposes. See Machamer and McGuire (*forthcoming*), Schmaltz 2008; Guéroult 1980: Ch. 14; Gabbey 1980; Hatfield 1979; Garber 1992: 70ff.; Gorham 2004.

[33] See Garber 1992: Ch. 3. The *Meditations* also provides a third argument, which Garber calls the argument from objective reality, but which does not reappear in the *Principles*.

of wax because in this argument Descartes asks us to consider how we can apply a general concept—a nature, a genus of things—to a particular bit of matter. Consequently, that bit of matter is a stand-in to all objects which may fall under that general concept. Although all the perceptible qualities of the wax change, there seems to be a single feature of it that remains; namely, that it is extended in length, breadth, and depth. What does this mean for our *idea* of this wax, the genus under which we understand it? Descartes presupposes that to the extent that we have a distinct idea of the wax, we have the *same* idea of it before and after heating—it is, after all, the *same* piece of wax. Thus, he argues, since all perceptible qualities of the wax change, those qualities cannot be elements of our distinct idea of the wax. Rather, insofar as our idea of the wax is conceived distinctly, that idea must solely be an idea of the feature of the wax that does not change; namely, that it is extended, or more properly, that it falls under the genus 'extension'.[34]

It is generally acknowledged that the argument does not succeed.[35] However, success and failure aside, it is clear that the argument cannot count as a deduction of the content of Descartes' concept of body from metaphysical considerations. Rather, the argument only goes to certify that an already established idea of extension is, in fact, the appropriate content for the concept of body. The structure of the argument, after all, is to take our pre-philosophical idea of a body and strip from it elements that do not pass a certain test. That test may be inspired by metaphysical considerations (i.e., that variable features of an object cannot be part of the nature of that object), but the test itself is not responsible for generating the content of the resulting idea. Rather, our vulgar idea of the wax, as obscure and confused as it initially is, must already contain within it the seeds of the geometrical conception of body endorsed by the conclusion to elimination argument.

But I am being unfair. Descartes did not intend the argument to be understood as a deduction of his concept of body, at least not in the *Meditations*. In that work, the argument goes to show that mind is perceived more distinctly than body, and perhaps to foreshadow Descartes' later discussion of body, but it is not intended to fix the idea of body. Descartes

[34]As Descartes explains later in the *Meditations*, this is a feature of the wax we had understand by our intellects, not perceive through the senses. In the language of general/particular, although we *see* particular shapes, we *grasp* that these shapes falls under the genus of extension.

[35]See Williams 1978: 214ff.; Garber 1992: 79–80, but also Rozemond 1998: 91ff. for a dissenting opinion regarding the success of the argument in the context of the *Principles*.

is clear about this in his reply to Hobbes' objection to the *Meditations*. Concerning the passage containing the elimination argument, he notes: "I was not dealing in that passage with the formal concept of the mind or even with that of the body" (AT VI 175, CSM II 124).

The case is both similar and different in the *Principles*. In the *Principles*, Descartes also asks the reader to consider the elimination of sensible qualities from body, but here the argument is explicitly offered in support of the claim that "The nature of body consists... simply in extension". Here, Descartes asks us imagine that "whenever our hands moved in a given direction, all the bodies in that area were to move away at the same speed as that of our approaching hands" (AT VIIA 42, CSM I 224). Clearly, he argues, we would never have any sensation of hardness in bodies, but it is unreasonable to suppose that bodies would thereby cease to be bodies. Thus, hardness cannot be essential to bodies. Similar reasoning is claimed to apply to, and thus to eliminate, all other sensible qualities. What is interesting about this argument is that although it is offered in support of the claim that the "nature of body consist... simply in extension", Descartes does not argues for why *this nature* remains after all sensible qualities are eliminated. The argument only concludes that "By the same reasoning it can be shown that weight, colour, and all other such qualities that are perceived by the senses as being in corporeal matter, can be removed from it, while the matter itself remains intact" (AT 42 CSM I 224 (L)). What matter itself consists of—that is, the relevant *nature*, the genus common to all perceptibles—is taken for granted. Consequently, if *this* is Descartes' argument for his conception of body in the *Principles*—as its title suggests—then the *content* of that conception—what we grasp when we grasp it—emphatically does not issue from metaphysical considerations. If this is the case, then perhaps we have already found the boundary between Descartes' physics and metaphysics exactly where we should have expected it to lie, in Descartes' conception of body, a conception whose content *insofar as the elimination argument is concerned* is not deduced from metaphysics, and so may be considered irreducibly physical.

But this might also be unfair. Although Descartes writes as if the elimination argument helps to *establish* his conception of body, technically speaking, by the time he introduces the argument in *Principles*, Part II §4, he has already determined the nature of body. In fact, without having previously done so, he could not prove the existence of material bodies as reified bits of extension in *Principles*, Part II §1, nor could he initiate the elimination argument with the hypothetical premise regarding the motion of actually existing bits of matter. In the overall structure of the *Principles*, the elimination argument only serves to defend an already established concept of material body from possible counter-arguments regarding the deliverances of the senses (Part II §4), or regarding the implication of Descartes' concept for the distinction between space and body (Part II §11). The elimination argument does not determine the content of the concept.

5.4.2 THE COMPLETE CONCEPT ARGUMENT (AND THE CONCEPT OF SUBSTANCE)

In the *Principles*, Descartes determines the content of the idea of body by means of the so-called complete concept argument (Part I §48ff.). The argument proceeds by classifying all of our ideas concerning "things" or "affections of things" (understood to refer to objects, qualities, attributes, modes, etc.) into two genera and determining, within each genus, the single idea that is presupposed by every other idea in that genus. Descartes does not independently argue for the existence and uniqueness of such an idea, but only claims that, in fact, we can find one such idea when reflecting on the conceptual presuppositions of ideas in a single genus. Descartes' two genera are the genus of ideas that pertain (*pertinent*) to mind and genus of ideas that pertain to body.[36] Not surprisingly, the idea of a *thinking*

[36] Although at this point in the *Principles* Descartes has yet to come to an official definition of body, it seems his ability to sort ideas into these classes depends only on the mind/body distinction drawn earlier in the *Principles*, a distinction which does not require the nature of both substances to be previously determined. Although the grounds for the distinction are beyond our scope, this is the short story: Descartes draws the distinction between "soul and body, or between a thinking thing and a corporeal thing" in Part I §8. He holds that we arrive at this distinction by reflecting on the character of minds, on our nature as thinking things as revealed by the *cogito* (*Principles* I §7). He writes:

> This is the best way to discover the nature of the mind and the distinction between the mind and the body. For if we, who are supposing that everything which is distinct from us is false, examine what we are, we see very clearly that neither extension nor shape nor local motion, nor anything of this kind which is attributable to a body, belongs to our nature, but that thought alone belong to it. (*Principles* I §8 Latin, AT VIIIA, CSM II 195)

Although this disconnected quote may suggest Descartes is already assuming that body is *res extensa*, he is not. Although he holds that extension, shape, motion and "anything of this kind" is *attributable* to body, *substance* turns out to be presupposed by all ideas of the first class (other than itself, of course) and the idea of an *extended substance* by all ideas of the second class.[37] Given this last claim, Descartes concludes that 'extended substance' is the complete concept of body, that is, that it represents all that ought to be represented about body in general.

We will spend the remainder of this section investigating how the concept of extended substance is presupposed by all ideas that pertain to body, and what this means for the relation of Descartes' account of body to his metaphysics. We'll see that the complete concept argument is actually comprised of two sub-arguments, one concerning the *essence* of body and one concerning the relationship of this essence to the overall concept of body as extended *substance*. This dual structure reveals that the content of Descartes' concept of body is not just that of extension—as the elimination argument may have suggested—but that of *substantial extension*.[38] In other words, the argument shows that the content of Descartes' concept of body is partly, but essentially, comprised of the metaphysical concept of substance. In this way, it serves to tie the concept of body directly to metaphysics. To see this, we must pay particular attention to the way in which the concept of substance plays a role in tracing the conceptual presuppositions of the class of ideas that pertain to body.

Let's begin with the concept of substance itself. Shortly after classifying all of our ideas into two genera, Descartes defines substance as "a thing which exists in such a way as to depend on no other thing for its existence".[39] Although the definition does not apply univocally to both God and created things (since God exists *simpliciter*, while created substances exist due to the creating and conserving power of God), Descartes holds that both sorts of

'body' here is taken in the ordinary sense of the word, and Descartes merely asserts that there seem to be a whole host of concepts that are commonly attributed to it, but that cannot be attributed to mind. For the purpose of revealing what is essential to mind and drawing the distinction between mind and body, this is sufficient.

[37]The argument also considers those ideas that have to do with the "close and profound" union of mind and body, but in the Latin edition these are considered as genera-straddling, not as constituting their own genus. These are foreign to our purposes here and are treated more fully by Descartes in *Principles*, IV, §189ff. The status of these idea in the French edition is more complex. See Garber 1992: 91–93.

[38]I owe much to Daniel Garber's and Peter Schouls' discussion of this point, Garber 1992: 88 and Schouls 1980: 122ff..

[39]In the scholastic-Aristotelian tradition, substance was also defined by its independence, but vis-à-vis *predication*. Curiously, Descartes had defined substance by means of predication in earlier work, but had abandoned this definition by the *Principles*. See in particular the Geometrical Appendix to the *Second Replies*. For a broad treatment of this issue, see Des Chene 1996: Ch. 4.

things may be considered substances.[40] In the French *Principles*, he further elaborates on the status of created things and clarifies the meaning of 'attribute' in relation to 'substance':

> In the case of created things, some are of such a nature that they cannot exist without other [created] things, while some need only the ordinary concurrence of God in order to exist. We make this distinction by calling the latter 'substances' and the former 'qualities' or 'attributes' of those substances (Part I, §51, AT IX 47, CSM I 210)

In other words, attributes (qualities, modes, etc.) depend for their being on the being of substance, and created substances depend for their being on God. This is what Descartes considers the "common concept" of substance. Although particular substances are understood to fall under this concept (*sub hoc communi conceptu intelligi*, §52), Descartes does not consider 'substance' a genus, but one of the terms that "extend to all genera of things" (*ad omnia genera rerum se extendunt*, §48).[41]

Descartes also notes that although attributes depend on substance for their being, a particular kind of substance (not the common concept 'substance') cannot be known without its attributes. That is, since existence "does not of itself have any effect on us", our knowledge of a particular kind of substance can only be derived from our knowledge of its attributes (Part I §52, AT VIIIA 25, CSM I 210).[42] Descartes' language is deceptive here. Although the claim that "a substance itself is known" by its attributes (*[substantia] ipsa cognoscatur*) seems to suggest that we know *what* a substance is by its attributes, Descartes' argument in this article only shows that we know *that* it is by its attributes. His argument rests on the claim that "nothingness possesses no attribute, that is to say, no properties or qualities", and so, "if we perceive the presence of some attribute, we can infer that there must also be present an existing thing or substance to which it may be attributed" (Ibid.). Clearly, the *existence* of a substance can be discovered in this way, but its *nature* or essence—what it is—cannot.

Descartes completes his account of substance by outlining how we can come to know the essence of a particular substance. In the very next article of the *Principles*, he writes:

> A substance may indeed be known [*substantia cognoscitur*, again] through any attribute at all; but each substance has one principal property which constitutes its nature and essence,

[40] See also AT VII 226, CSM II 159 and the importantly different AT VII 161, CSM II 114.
[41] See Chappell 1997 for a comprehensive treatment of Descartes' classification scheme.
[42] See also *Reply* to Arnauld, AT VII 222, CSM II 156.

> and to which all other properties are referred (*referuntur*). (Part I §53, AT VIIA 25, CSM I 210, original emphasis)

The principal property of a substance deserves to be called its "essence", according to Descartes, because although we may be in possession of an entire class of ideas pertaining to that substance, all such ideas bear a special, asymmetric relation to the principal property—they must be "referred" to it. Descartes explains this 'referring' relation by way of an example. In the case of body, the principal property is extension, and, according to Descartes:

> Everything [other than extension] which can be attributed to body presupposed extension. . . For example, shape is unintelligible except in an extended thing; and motion is unintelligible except as motion in an extended space. . . By contrast, it is possible to understand extension without shape or movement. . . (Part I §53, AT VIIA 25, CSM I 210–211)

The example shows that the special 'referring' relation all properties bear to the principal property concerns the way in which we understand these properties. The gist of the underlying argument, insofar as body is concerned, is this: When we reflect on the ideas we associate with body, we discover that most of them are "unintelligible". They are unintelligible, according to Descartes, because we cannot clearly perceive them independently of other concepts. Rather, in order to perceive them clearly, we must perceive them as part of another, more complete concept. That concept is 'extension' and because whenever we think of anything that pertain to body we must think of it, it must constitute the primary attribute and essence of body.

Of course, this argument, which constitutes the first phase of the complete concept argument, is rather telegraphic. Luckily, Descartes offers some illuminating remarks in correspondence and other writings.[43] In the *Fourth Replies*, for example, he details why certain ideas cannot be clearly understood without others. He writes:

> [A]lthough a genus can be understood without this or that specific differentia, there is no way in which a species can be thought of without its genus. For example, we can easily understand the genus 'figure' without thinking of a circle. . . But we cannot understand any specific differentia of the 'circle' without at the same time thinking of the genus 'figure' (AT VII 223, CSMK 157)

[43]See Schouls 1980: 122ff.. The story I am about to sketch the tip of a complex theory of meaning and reference. For its details, see Machamer and McGuire (*manuscript*), Chapter 4.

The picture painted here is that the 'referring' relation of the *Principles* is nothing other than the genus-species relation, exactly as Descartes' initial division of ideas into two genera should have suggested. However, Descartes makes explicits here the claim that most of our ideas concerning body are in fact *species* of a general idea, the idea of extension. In the original division into genera, such ideas were only said to "pertain" to body. At any rate, since we cannot understand a species without reference to its genus, clearly perceiving any of these ideas is impossible without at the same time perceiving the genus to which they belong. In this way, if we follows the chain of ideas from the specific to the more and more general (say, from 'cube' to 'three-dimensional figure', to 'extended shape', etc.) we see that understanding any of our ideas concerning body in a clear fashion requires that we conceive of the single highest genus to which they all belong, the genus of extension. Since this is the only idea concerning body that can be understood both clearly and distinctly, it represents for Descartes the essence of body.

Two points should be made about this argument. First and in regards to the demarcation problem, it is clear that this argument does not 'generate' (to speak loosely) our idea of the essence of body—that is, extension—from metaphysical principles. Rather, like the elimination argument, it takes an already known concept of extension—an idea naturally found in the class of ideas that pertain to body—and shows that it is the appropriate idea for the essence of body. In this regard, the complete concept argument may ultimately be more convincing than the elimination argument, but the former is no different than the latter in that it assumes that we are already in possession of the concept of extension.

Second, and more importantly, this argument does not depend or impact on Descartes' definition of substance as a thing that is capable of independent existence. By reflecting on the genus-species relations found in our ideas concerning body, Descartes shows that our ideas presuppose the essence of body, but, thus far, his argument does not reveal what this presupposition has to do with the notion of an independently existing thing. At this stage of the complete concept argument, we still have not arrived at the idea of an *extended substance*. This last, crucial part of the *Principles* complete concept argument is intertwined with the genus-species claim in the *Principles* itself, but it is made explicit in an oft-quoted letter to Gibieuf (19 January 1642). There, Descartes explains what he means by an "incomplete idea", the sort of idea that is unintelligible on its own. He writes:

> To tell whether my idea has been made incomplete or inadequate..., I merely look to see whether I have derived it...from some other, richer or more complete idea which I have in myself. This intellectual abstraction consists in my turning my thought away from one part of the contents of this richer idea...Thus, when I consider shape without thinking of the substance or the extension whose shape it is, I make a mental abstraction...For I see clearly that the idea of the shape in question is joined in this way to the idea of the corresponding extension and substance, since it is impossible to conceive a shape while denying that it has an extension...(AT III 473–474, CSMK 201–202)

Thus far, we might be tempted to read the genus-species relation discussed in the *Fourth Replies* into Descartes' claim that the concept of shape is incomplete without the concept of substance. However, Descartes is explicit in the *Principles* that 'substance' does not constitute a genus, but a concept that 'extends' (*extendere*) to different genera (Part I, §48, VIIA 23, CSM I 208). The highest genera are only the things that concern body and the things that concern mind. Thus, the claim that shape, or more generally extension, is not fully conceivable without substance must rest on different grounds. Descartes' addresses these explicitly at the conclusion of the passage just quoted. He writes:

> It is impossible to conceive an extension while denying that it is the extension of a substance. But the idea of a substance with extension and shape is a complete idea, because I conceive it entirely on its own (Ibid.)

This is the final step of the complete concept argument, and it brings into view how the idea of substance enters into Descartes' definition of body. We see here that the chain of species and genera of the *Fourth Replies* cannot terminate in the concept of bare extension. Rather, even when we conceive extension, we conceive an incomplete concept. This is because, according to Descartes' definitions of substance and attributes, the attribute of extension must itself be supposed to exist *in a substance*. Consequently, for Descartes even the idea of extension is incomplete, since it is drawn from "a richer or more complete idea" we have. Put in terms of clearness and distinctness, since Descartes holds that a truly complete concept is one that can be conceived independently of all others in all respects—i.e., one that can be perceived distinctly—and since he defines an attribute as a 'thing' that is dependent on substance, it follows that extension cannot be conceived of distinctly with respect to its existence. Consequently, it is an incomplete concept. And *a fortiori*, although extension *simpliciter* is an incomplete concept, the idea of an extended substance is not.

We can now see how Descartes' concept of body embodies both metaphysical and non-metaphysical notions. Body, according to Descartes, is an extended substance, an extension capable of independent existence. And although the concept of extension is not *per se* metaphysical, the concept of an independently existing thing is. This last component of the concept of body is indispensable, since without it the very concept of body would be incomplete and thus "unintelligible". Of course, I am assuming here that the concept of extension falls under the domain of physics. One could argue that this assumption is wrong, i.e., that the concept of extension is itself a metaphysical concept, on par with the concept of substance. This position is entailed by any interpretation on which the purpose of metaphysics is to reveal essences, and extension, being the essence of matter, thus becomes a metaphysical concept.[44] Under this interpretation, it is the role of extension vis-à-vis the remainder of our ideas (i.e., that it is the attribute to which all ideas that concerns corporeal bodies must refer) that makes the concept metaphysical. Note, however, that this interpretation would only strengthen my current case, since it would entail that one cannot demarcate Cartesian metaphysics and physics by their treatment of the concept of matter. Despite this, the interpretation is unsatisfactory because it cannot make sense of Descartes' statements to the effect that his physics deals with material things (See §5.3). As I noted earlier, it is the separation of metaphysics and physics according to the range of topics they cover (i.e., their content, not function) that I am attempting to understand here, so positions like the one just mentioned are orthogonal to my concerns.

At any rate, we cannot draw a sharp line between Descartes' metaphysics and his concept of body. At best, we can claim that Descartes' concept of body is partly comprised of metaphysical content (i.e., Descartes' idea of substance) and partly comprised of another sort of content (i.e., Descartes' idea of extension), and so is both continuous and discontinuous with his metaphysics. In fact, its seems that *res extensa* is a concept that straddles boundaries in the Cartesian system. It is first established in Part I of the *Principles* (and the *Meditations*!) as part of a metaphysical deliberation on the nature and interrelations of ideas innate to the human mind. Nevertheless, it is also discussed in propositions 4-23 of Part II of the *Princi-*

[44] See, e.g., Secada 2000.

ples as part of an inquiry into the physics of the created universe, and had appeared years earlier in Descartes' earliest exposition of 'his physics' in *Le Monde.* But there is more to Descartes' physics than just the concept of matter. As Descartes' himself claims, "all of the dispositions which we perceive [in matter]... result from the movement of its parts" (*Principles* Part II, §23, AT VIII 52). It is this central part of Cartesian physics—motion—that we now turn to consider.

5.4.3 MOTION

Descartes defines motion in the *Principles* after explaining that it is the central concept of his physics. Motion's central role follows from the fact that Descartes' minimalist concept of matter cannot by itself allow for change in the created universe:

> For although our minds can imagine divisions [in matter], this alone cannot change matter in any way; rather, all the variation of matter, or all the diversity of its forms, depends on motion. Further, this seems to have been noticed by Philosophers everywhere; because they have said that nature is the principle of motion and rest. And by 'nature' they then understood that by means of which all corporeal things become as we experience them to be (*Principles* Part II, §23, AT VIII 52–53).

In this passage, Descartes wants to draw attention to the supposedly deep agreement between his conception of physics and that of the "Philosophers". Both, after all, have motion at their center. This is an important point, since a scholastic-Aristotelian reader would have taken this as a signpost: 'Descartes' physics concerns the same objects as mine, those that are liable to change'. However, as Descartes makes clear right away, the Cartesian notion of motion and change is radically different—not in its central function, but in its content—from that of the Philosophers. For Descartes, motion, philosophically speaking, is "the transference of one part of matter or of one body, from the vicinity of those bodies immediately contiguous to it and considered as at rest into the vicinity of others" (*Principles*, Part II §25, AT VIII 53–54). This definition is introduced immediately after a quick dismissal of the "vulgar" definition of motion—"the action by which some body travels from one place to another"—on the ground that it leads to contradictory ascription of 'motion' and 'rest' to particular bodies (*Principles*, Part II §25, AT VIII 53).

As has been recognized by a variety of commentators, the "philosophical" definition of motion has both metaphysical motivations and metaphysical implications.[45] Its primary metaphysical motivation, as Garber 1992: Ch. 6 points out, is to render motion a proper mode of body, something regarding which there is a non-arbitrary matter of fact. It is the arbitrariness implicit in the vulgar definition to which Descartes explicitly objects, and thus, his main *desideratum* for a proper definition is that it avoid such arbitrariness.[46] Apart from this, the definition has obvious implications for questions of individuation, non-teleological change in the Cartesian universe, and to a host of issues concerning the causes of motion and their ontological status.

However, I'd like to point out that in the *Principles* Descartes' strategy for establishing the concept of motion does not involve deducing the content of the concept—i.e., that motion is "transference of one part of matter..."—from any metaphysical principles. In the arc of that work, Descartes entertains two (and only two!) options for the definition of motion and chooses the philosophical one over the vulgar one on the grounds mentioned above. In other words, Descartes assumes that there are only two candidates for "motion" and argues in favor of the philosophical one on the grounds that it avoids relativism. Not only does Descartes not show us anything like a derivation of the content of the philosophical concept, he doesn't even show us, assuming the content already given, how he has arrived at that concept as a definition of *motion* (as he did, say, for the concept of extension as the primary attribute of body!). Descartes merely invokes the philosophical definition (using the vulgar definition to explicate its terms) and, *having done so*, argues that it is the appropriate one. The definition itself, however, seems to come *ex nihilo*.[47] Of course, this should not be entirely surprising given what we have said at the close of the previous section (§5.4). Descartes believes that

[45]See, e.g., Blackwell 1966, Guéroult 1980, Gabbey 1980, Garber 1987, 1992, Des Chene 1996, Sowaal 2001, Arthur 2004, Gorham 2005, Machamer and McGuire 2006, Schmaltz 2008. This literature is important because it does away with the caricature of Descartes' definition as an *ad-hoc* band-aid solution to the problem of terrestrial motion in Copernicanism, a band-aid applied after Galileo's condemnation for the sole purpose of avoiding censure by the Church. See Henry More's (1662) *Preface General*.

[46]Later criticisms of Descartes' physics would agree that the relativism of the vulgar definition ought to be avoided, but argued that the philosophical definition is vulnerable to a similar charge. Newton's *De Gravitatione* is perhaps the best case in point. See Newton 2007 and DiSalle 2006: Chap. 2.5–2.6.

[47]This is not to say that there are no *historical* precedents for Descartes' definition. In fact, Descartes definition shares many features with the scholastic-Aristotelian definition of *motus localis*. My point is merely that in the argumentative chain of the *Principles*, the definition is presented, not reasoned to.

the concept of motion, like the concept of extension, is just one of those concepts innately available to the human mind. We need no procedure for establishing its content because that content is always and already with us. In fact, Descartes confidence in the immediacy of the concept of motion to our cognitive faculties is somewhat of a staple in his writings. In *Le Monde*, for example, he writes:

> [T]he nature of the motion that I mean to speak of here is so easily known that even geometers, who among all men are the most concerned to conceive the things they study very distinctly, have judged it simpler and more intelligible than the nature of surfaces and lines. . . (*Le Monde* 26, AT XI 39)

Of course, motion in *Le Monde* is entirely different than motion in the *Principles*, but Descartes' attitude towards it is similar: Nothing is simpler than motion, Descartes suggests, and thus we do not reason to, only from, it. Similarly, in the *Regulae*, motion was conceived as a simple nature, and thus known immediately and through itself, not through any other concepts (*Regulae*, Rule 12).

Nevertheless and despite the immediacy of the concept, Descartes does believe that it has several note-worthy features that are not *prima facie* obvious but are are crucial for our understanding it.[48] We have already mentioned the first: that motion is a mode. This fact is significant because it signals that there is a metaphysical backing for Descartes' concept of motion, just as there is metaphysical backing for his concept of *res extensa*. First, since motion is a mode, it is incapable of independent existence and must exist as mode *of* some substance. In the case of motion, Descartes had earlier noted (even before offering the definition of motion!) that the concept must be "referred" to the primary attribute of extension, and so its relevant substance is *res extensa* (*Principles* Part I §53, Part II §25, 27). This implies not only that the definition of motion cannot be understood without the concept of extension (as should be obvious, since there is no sense to the notions of vicinity and contiguity without extension), but that it cannot be understood without the concept of an *existing, substantial* extension. In §36, Descartes will use the inherence of motion in substantial extension to argue that since God is responsible for continually underwriting the existence of matter, he is also responsible for the existence of all motion (since it exists in

[48] Although much can be (and has been) said about these, I'll merely review them here (the reason for skirting nuances will become clear in the following section).

matter), and thus that we can learn about the motions that exist in our world by reflecting on God's action. This feature of motion ultimately warrants Descartes' move from considering motion as it is in itself, independently of its cause, to considering its cause. In the secondary literature, this feature has also been used to explain aspects of the Cartesian conception of motive force through the existence and duration of the material substance of which it is a mode.[49]

What does motion's modal status mean for our demarcation problem? As in the case of *res extensa*, it seems that the content of Descartes' concept of motion—what, as Gassendi put it, we grasp when we grasp the concept—cannot be entirely divorced from metaphysics (being a mode) but cannot be entirely metaphysical (being about the transference of bodies). Indeed, it has both a metaphysical and a non-metaphysical component. Has Descartes "deduced" it from metaphysics? As in the case of *res extensa*, there is a sense in which he has and a sense in which he hasn't. Certainly, metaphysics seems to provide the criteria that lead Descartes to endorse his definition of motion as the correct one. However, what we grasp when we grasp the concept of motion certainly does not emerge entirely from metaphysical considerations. Thus, for someone wondering how Descartes can possibly claim to deduce physical truths from premises devoid of content about the changing, natural world, the "easy solution" of showing that the metaphysical truths themselves are responsible for generating physical content is closed off. Of course, I should emphasize (as I have throughout) that this way of looking at the disciplinary relations implicit in the Cartesian system is by no means the only one. Considering Cartesian metaphysics and physics in light on the classificatory and justificatory functions of the former does not yield a demarcation of the sort I am concerned with. Nevertheless, the problem is not a vacuous one since it is likely that it would have been felt by someone looking at Descartes' system through scholastic-Aristotelian eyes, i.e., using the cluster of concepts discussed in Chapter 2 and 3. In particular, someone who expects to find sciences individuated according to the *objects criterion* would have a hard time seeing how truths about the objects of physics can drive from truths about the objects of metaphysics. The question, to *that* a reader, would indeed have been how Descartes can possibly claim to deduce physical truths from premises devoid of content about the changing,

[49]See Gabbey 1980, Guéroult 1980, Schmaltz 2008.

natural world and so the demarcation problem would have been a live concern. Would such a reader be able to find a possible solution within Descartes' system? I'd like to answer in the affirmative. Here is my proposal: a scholastic-Aristotelian reader would have been able to read the relation of physics to metaphysics in Descartes on the model of subalternation.

To see this, we need to turn to the laws. Before doing so, however, I need to point to two other features of Descartes' concept of motion that are significant for our purposes; namely, that motion has a quantity and that it has a direction. Although Descartes doesn't use the term "quantity of motion" until §36 (the beginning of his discussion of the causes of motion and the derivation of the laws of nature), he clearly assumes it in the preceding discussion of the definition of motion. When considering the relation of a moving body to its immediately contiguous neighbors, Descartes writes that:

> Thus, if we wish to attribute to movement a nature which is absolutely its own, without referring it to any other thing; then when two immediately contiguous bodies are transported, one in one direction and the other in another (*unum in unum, aliud in aliam partem transferuntur*), and are thereby separated from each other; we should say that there is as much movement in the one as in the other (*tantundem motus in uno quam in altero*) (*Principles* Part II §29, AT VIII 56).

Notice the use of "as much…as" [tantundem…quam]: it indicates that to the extent that there is motion in one body, there is motion in the other. Although Descartes' does not make it explicit, for this comparison to hold there must be some means of juxtaposing motions and finding them equal; namely, a measure. Of course, a few passages later Descartes makes it fully known that motion has a measure; namely, its quantity. However, I want to point out that he very much assumes that the existence of this measure follows from the definition itself: it is a feature of motion that follows from motion's having a nature "which is absolutely its own", not a feature that is incumbent on the causal nexus in which it is embedded.

Descartes also notes that motion has a direction. In §32 ("How, properly understood, the single movement peculiar to each body may also be regarded as multiple"), he writes (See Figure 6):

> We can even consider this single movement which is peculiar to each body as equivalent to several movements…for one can imagine any line whatever, even a straight one…to have been described by innumerable diverse movements. For example , if at the same time as the AB moves towards CD, its point A moves closer to B; the straight line AD (which will be described by the point A), will depend no less on the two [rectilinear] movements

(*motibus rectis*) of A toward B and of AB toward CD... (*Principles* Part II §32, AT VIII 58)

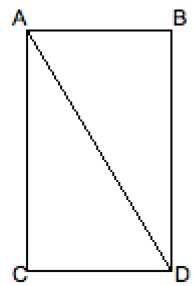

Figure 6: *The Composition of Motions*

Certainly, the composition of motions would not make sense unless each individual motion had a direction (from A to B, say), something that is very clearly assumed in Descartes' diagram as well as his exposition.

These features—the quantity, direction, and uniqueness—of motion, will each play a part in Descartes" deduction of the laws of nature.

5.5 SUBALTERNATION IN ACTION: THE DEDUCTION OF THE LAWS OF NATURE

Descartes' laws of nature govern how motion is distributed in the Cartesian world and, given motion's central role in accounting for the "diversity of forms" of matter, are thus the centerpiece of Cartesian physics. Their deduction has been oft-discussed.[50] Much of this discussion is concerned with fleshing out the hidden details of Descartes' brief exposition, with finding a particular conception of force or God's action implicit in the mechanics of the deduction, and with fixing the proper interpretation of the laws themselves (particularly the third law). While these are worthwhile concerns, they are not my own. I'd only like to

[50] See, e.g., Gorham 2005, Des Chene 1996: Chap. 8, Garber 1992: Chaps. 9, Nadler 1990, Osler 1985, Guéroult 1953, 1980, Hatfield 1979, Williams 1978, Schouls 1972, Blackwell 1966.

show that, *independently of the way its details are cashed out*, the deduction has an overt structure that fits the mould of a demonstration in the subalternate sciences.[51]

Having drawn the distinction between motion considered in itself and motion considered through its cause and having examined the nature of motion taken in itself, Descartes explains the cause of motion, which is two-fold. He writes:

> [First there is] the universal and primary one [*universalem & primariam*], which is the general cause [*causa generalis*] of all movements in the world; and then the particular [*particularem*] ones, by which individual parts of matter acquire movements which they did not previously have (*Principles* Part II §36, AT VIII 61).

These particular causes will turn out to be the laws of nature, the laws determining the actual motions of bits of matter. How does Descartes arrive at them? Given that God is the primary and general cause of change in Descartes' universe, Descartes believes he can discover the properties of the secondary and particular laws of nature by reflecting on God's ultimate causality. He writes:

> [We] understand that it is one of God's perfection to be not only immutable in His nature, but also immutable and completely constant in the way He acts... From this it follows that it is completely consistent with reason for us to think that, solely because God moved the parts of matter in diverse ways when He first created them, and still maintains all this matter exactly as it was at its creation, and subject to the same laws as at that time; He also always maintain in it an equal quantity of motion (Ibid.).

This, Descartes' conservation principle, is the first result of his consideration of the immutable action of God. Although it is not one of his three laws, its derivation presages that of the laws and is later more directly involved in the (supposed) justification of the third law. Despite its separate status, Descartes' justification of the conversation principle mirrors exactly the justification he provides for his three laws:

> [Law 1:] [F]rom this same immutability of God, we can obtain knowledge of the rules or laws of nature, which are the secondary and particular causes of the diverse movements which we notice in individual bodies. The first of these laws is that... (*Principles* Part II §37, AT VIII 62)

[51] Thus, although I am skirting the standard issues involved in interpreting the laws—creation vs. concurrence vs. sustenance, God's underwriting of the existence of bodies vs. his causing of motion, the nature of God's causality (that it is efficient and how), the ontological status of force, the relation between the measure of God's supposed action (vis-à-vis the force of motion) and the measures of the observable properties of matter—my point is precisely that my interpretation is not dependent on any of these issues. In a sense, I believe we might have missed the forest for the trees.

> [Law 2:] The second law of nature... This rule, like the preceding one, results from the immutability and simplicity of the operation by which God maintains movement in matter (*Principles* Part II §39, AT VIII 63)

> [Law 3:] [T]he second part [of the third law] is proved by the immutability of God's manner of working in always uninterruptedly maintaining the world by the same action by which He created it. (*Principles* Part II §42, AT VIII 66)

The laws themselves are:

> [Law 1:] [E]ach thing, in so far as it can (*quantum in se est*), always continues in the same state; and thus what is once in motion always continues to move (*Principles* Part II §37, CSM I 240, AT VIII 62)

> [Law 2:] [A]ll motion is in itself rectilinear; and hence any body moving in a circle always tends to move away from the centre of the circle which it describes (*Principles* Part II §39, CSM I 241 AT VIII 63)

> [Law 3:] [I]f a body collides with another body that is stronger than itself, it loses none of its motion; but if it collides with a weaker body, it loses a quantity of motion equal to that which it imparts to the other body (*Principles* Part II §40, CSM I 242, AT VIII 242)

The first law governs the states of motion and rest of individual bodies, the second law govern the direction of individual moving bodies (and, in its exposition, gives rise to the notion of "determination" which is only fully articulated in the exposition of the third law), and the third law governs exchanges of motion (both according to quantity and the newly articulated notion determination) of colliding bodies.

What is striking is that each law is deduced from precisely the same principle: God's immutability.[52] Of course, the particulars of each deduction highlight a different way in which God's immutability manifests in the world of motion: for instance, the first law focuses on the notion that God maintains modes of body and the second law focuses on the notion that God maintains motion 'in an instant' (and thus that he maintain an instantaneous "determination" *to* motion).[53] However, *the basic structure of each deduction is the same.* In each deduction, God's immutability, combined with some features of motion and perhaps

[52]In fact, Descartes takes this justification to be so compelling that he even the third law as *evidence* for the immutability of God (Principles II §42, AT VIII 66)

[53]The grounds for the third law, apart from the immutability of God, are so unclear that Daniel Garber writes that "the law that actually governs the exchange of motion [i.e., the third law]... is offered no justification at all" (Garber 1992: 292).

some auxiliary principles regarding the nature of God's immutable action, yield the law in question. As Denis Des Chene (1996: 324) writes, "the formal condition of immutability yields no natural law except in conjunction with some further condition", and so, in each case, God's immutability is conjoined with some further condition(s). These conditions, again, are physical facts about the nature of motion and further metaphysical facts about the nature of God's action. For example, to derive the first law, Descartes first suggests that God's immutability is to be cashed out in his preservation of *states*—which, as Garber explicates, are *modes* of bodies.[54] Descartes provides the example of a body's shape:

> [I]f a particular piece of matter is square, we can be sure without more ado [given the constancy and immutability of God's action] that it will remain square for ever, unless something coming from outside changes its shape. (*Principles* Part II §37, AT VIII 62)

Then, he applies the preservation of modes of bodies to a particular mode: motion. Given that he has already shown that motion is uniquely assignable to any body, the application is straightforward:

> If it is at rest, we hold that it will never begin to move unless it is pushed into motion by some cause. And if it moves, there is equally no reason for thinking it will ever lose this motion of its own accord and without being checked by something else. Hence we must conclude that what is in motion always, so far as it can, continues to move (*Ibid.*).

This last statement is, of course, the first law.

The second law works in much the same way. Once again, Descartes begins with the immutability of God's action. In this case, however, he also brings to the table a certain conception of God's action in time, namely, that God must underwrite the existence of the natural world at each instant, independently of all other instants:

> This rule, like the preceding one, results from the immutability and simplicity of the operation by which God maintains movement in matter; for He only maintains it precisely as it is at the very moment at which He is maintaing it, and not as it may perhaps have been at some earlier time.... (Ibid., §39)

To this conception of God's action Descartes adds the fact that the definition of motion entails that a given motion has a direction. However, since no motion takes place at an instant (as God's action does), at an instant such a direction corresponds to a body's "determination". That determination is unique since assignments of motion are themselves unique:

[54] E.g., Garber 1992: 214.

> Of course, no movement takes place in an instant; yet it is obvious that every moving body, at any given moment in the course of its movement, is determined to continue that movement in some direction in a straight line.. . . (Ibid.)

Needless to say, there is a good deal that is packed into this single claim, but, insofar as Descartes' exposition of the deduction of the second law is concerned, that complexity does not alter the structure of the deduction. Given a body's determination to move in a particular direction, God's immutable action guarantees that, unless interfered with, a body will *actually* move in that direction. That is, God will act to conserve the same determination in successive instants of time, and so bodies, unless interfered with, trace out straight lines. In the derivation of the second law, the immutability of God's action is applied to physical facts about the uniqueness, direction and determination of motion.

Finally, in justifying the conservation principle (my proxy for the third law, since the justification for that law is so sketchy), Descartes focuses on motion's quantity. The inference here seems to be immediate, and assumes only that God acts immutably and that motion admits of quantity:

> [S]olely because God moved the parts of matter in diverse ways when He first created them, and still maintains all this matter exactly as it was at its creation, and subject to the same laws as at that time; He also always maintain in it an equal quantity of motion(*Principles* Part II §36, AT VIII 61)

Once again, God's immutability is applied to a certain aspect of motion, its quantity.

In each of the three cases, a metaphysical truth (or several metaphysical truths) about God's action is (or are) applied to the properties of motion. This application is licensed by God's being the ultimate cause of motion and yields the laws of nature, the very laws that form the centerpiece of Descartes' physics. Note, moreover, that although Descartes' justification of the third law—the law governing exchanges of motion between bodies of various sizes—is lacking in sufficient detail to allow for a reconstruction, any possible reconstruction would have to involve both the properties of *motion* and the properties of *matter*, since in that law the quantity of a body's motion is measured by both its speed and volume. Volume, of course, follows from a body's extension, not its motion. Thus, what I say above about the conservation principle, the first and second laws and motion applies *mutatis mutandis* to the third law, motion and matter.

My argument regarding subalternation should be transparent at this point. I am claiming that a scholastic-Aristotelian reader would have been able to read the relation of physics to metaphysics in Descartes on the model of subalternation. That is, he would have been able to find enough resources in the *Principles* to claim that Cartesian physics "borrows" principles from Cartesian metaphysics and applies them in its own domain; that is, that *physics is subalternated to metaphysics*, that is is an *applied metaphysics*. As Chapter 3 showed, this is a claim a scholastic-Aristotelian reader would have been familiar with. On this understanding of the *Principles*, the problem of how physical truths can be deduced from metaphysical premises does not arise because in the application of metaphysical premises to the physical domain, metaphysical premises are, by the nature of subalternation, conjoined with content from physics. What does physics borrow from metaphysics? It borrows principles concerning God's immutable action. Those principles, however, are applied to features that are definitional of the physical domain, namely, matter and motion. Thus, although Descartes' way of arriving at the concepts of matter and motion would not have counted as a deduction of physics from metaphysics for a scholastic-Aristotelian (since each concept, as I showed in §5.4, is not reducible to metaphysics), the deduction of the laws of nature would have, since (as we've seen in Chapters 2 and 4) it has the form of deductions in the subalternate sciences! In this way, it is possible to hold on to the two apparently contradictory statements with which this chapter began: namely, that Cartesian metaphysics and physics are distinguished by their content and that they form a continuous deductive whole.

To review, in this chapter, I tried to read the *Principles* through Aristotelian eyes. I began with the assumption that Cartesian physics and metaphysics must be distinguished according to their content. This assumption was natural in scholastic-Aristotelianism (as discussed in Chap. 2 and 3), and I tried to argue that it is also suggested by many of Descartes' own statements. Taking this assumption alongside Descartes' proclamations to have deduced physics from metaphysics, I suggested that a demarcation problem arrises. Given that the subject-matter of metaphysics, according to Descartes and his contemporaries, comprised either the general features of all being and/or features of particular disembodied beings, one might inquire how the truths of physics can be deduced from metaphysical premises that are themselves devoid of content about the changing, natural world. For much of this chapter, I tried to show that an easy solution to this problem—i.e., a demonstration that in the *Principles* physical content *does in fact* emerge from metaphysical principles—is untenable. And although Descartes does motivate his concepts of extension and motion by variety of metaphysical criteria, the *content* of those concepts is not in any way generated from metaphysical principles. Rather, those concepts are simply assumed by Descartes, doubtlessly because they are innate to the human mind.

In order to save the notion that the truths of physics are deduced from metaphysics, I suggested that we read the deduction of the laws of nature on the model of subalternation. On this model, the principles of one domain are applied to another, and through this application license claims about the latter domain. As I showed in Chapter 3, Descartes' contemporaries would have been familiar with the idea that metaphysics is related to physics in this way—i.e., that metaphysics subalternates physics—and perhaps would have even expected to find such a claim in a work of comprehensive scope such as the *Principles*. Finally, I claimed that Descartes' deduction of the laws of nature is in fact amenable to such an interpretation. That is, I showed that the particular demonstrations that lead to Descartes' laws of nature fit the model of a demonstration in a subalternate science, examples of which were studied in Chapter 2 and 4. This is a more stringent view of Cartesian deduction that the one I recommended when rejecting the *logical containment* assumption. However, we do not need endorse a weaker view in order to make sense of Descartes' derivation of physics from metaphysics.

5.6 CONCLUSION AND CIRCUMSTANTIAL EVIDENCE

To close this chapter, I'd like to provide some circumstantial evidence to the effect that Descartes was well aware of the concept of subalternation and might have conceived the relationship between his metaphysics and physics in its light. I do not mean to claim that this evidence is ultimately compelling. However, I do believe it is suggestive. To begin with, given Descartes' statements regarding Eustachius' *Summa* (reviewed in Chapter 3), we can confidently state that in the period preceding the composition of the *Principles*, Descartes would have been familiar with the idea that metaphysics subalternates the other sciences. However, this is not the only occasion Descartes had to entertain the notion of subalternation and the host of philosophical issues surrounding the mixed-mathematical sciences. In fact, it is highly likely that he first encountered them at La Flèche. The evidence here is straightforward. Although students at La Flèche did not read Aristotle in the original, they read the commentaries of Francesco Toledo (1532-1596), Pietro da Fonseca (1528-1599) (both professors at the *Collegio Romano*) and likely the Coimbra Commentaries on the *Categories*, *De Interpretatione*, sections of the *Prior* & *Posterior Analytics*, the *Topics* (first year of higher studies), the first eight book of the *Physics*, *De Caelo* (second year), the first two book of *De Generatione* (both second and third year), *De Anima*, and *Metaphysics* (third year). Not only does this list contain all the *loci classici* for the commentary tradition on mixed-mathematics (as discussed in Chapter 2), the requirement to examine the major elements of this tradition was written directly into the *Ratio studiorum*, the plan for Jesuit education. Of the roughly 440 word description of the philosophy curriculum, about an eighth was given to the requirement that:

> In order to give the whole second year to physical matters, he [the lecturer] should begin a fuller discussion of science at the end of the first year and in it he should include the major topics of the introduction to physics, such as the *divisions of science, abstractions, speculative and practical science, subordination, the difference of method in mathematics and physics, which is treated by Aristotle in the second book of the Physics*, and finally what Aristotle says about definition in the second book On the Soul.(Society of Jesus 1599: emphasis added).

Clearly, such an emphasis implies that Descartes could not have completed his education without encountering the ideas we discussed in previous chapters.

More significantly, Descartes seems to have been highly influenced by these ideas. In the *Regulae*, we get perhaps the most direct evidence that the notion of subalternation played a key role in Descartes' intellectual development. In an autobiographical note, Descartes wrote regarding his search for a *universal mathesis*:

> I began my investigation by inquiring what exactly is generally meant by the term 'mathematics' and *why it is that, in addition to arithmetic and geometry, science such as astronomy, music, optics, mechanics, among others, are called branches of mathematics*... I came to see that the exclusive concern of mathematics is with question of order or measure and that it is irrelevant whether the measure in question involves numbers, shapes, starts,

> sounds, or any other object whatever. This made me realize that there must be a general [*generalem*] science which explains all the point that can be raised concerning order and measure irrespective of the subject-matter. . . *How superior it is to these subordinate [subditis] sciences in both utility and simplicity* is clear from the fact that it covers all they deal with, and more besides; and any difficulties it involves apply to these as well, where as their particular subject-matter [*particularibus objectis*] involves difficulties which it lacks (Rule 4, CSM I 19, AT X 378, emphasis added).[55]

If Descartes' autobiographical sketch is true, he came to the idea of a *universal mathematics* by reflecting on the relation of the mixed mathematical sciences to higher mathematical disciplines. However, as the standard story goes, he also came to the idea of a universal *method* by reflecting on his science of universal mathematics.[56] Thus, there is reason to believe that when Descartes proclaimed in the *Regulae* that "all [sciences are] interconnected and interdependent", his conception of the interconnectedness and interdependence—that is, unity—of the sciences was guided by the ideas that the principles of a master, subalternating science may be applied to a variety of domains and thus result in the variety of particular, subalternated sciences.

The same idea is also alluded to in the *Discours*. In another autobiographical sketch, Descartes writes that concerning his quest for method:

> [O]bserving that the principles of [the other] sciences must all be borrowed (*empruntés*) from philosophy, in which I had not yet found any certain ones, I thought it essential that before anything else, I try to establish some principles of philosophy (*Discours* Part II, AT VI 22).[57]

As we saw in Chapter 2 and 3, a subalternated science was often said to "borrow" principles from its higher, subalternating science.

Does this evidence, scattered as it is and taken primarily from Descartes' early career, demonstrate that he had intended the relationship of physics and metaphysics in the the *Principles* to be understood on the model of subalternation? Certainly not. It does, however, suggest that he was familiar with the idea. Moreover, if, as I argued earlier in this chapter,

[55] CSM translate "*particularibus objectis*" as 'particular subject-matter'. However, given what we have said in Chapter 3 regarding the *objects criterion*, it should be clear that this translation distorts the notion that sciences are defined by their objects. Descartes here merely repeats what he has been taught in school.

[56] See, for example, Schuster 1977.

[57] Once again, I have not followed the standard CSM translation here since it renders *empruntés* as "derived", an unnatural translation given the availability of a french cognate.

the mechanics of the derivation of the laws of nature in the *Principles* are amenable to being *interpreted* according to the subalternation relation, perhaps I've provided a plausibility argument—an argument not only about how Descartes could have been read by an educated Aristotelian (my primary intention), but how he might have wanted to be read.

6.0 CONCLUSION: THE UNITY OF SCIENCE AND THE GEOMETRICAL MANNER

> [T]he general notion that philosophy... can and should be organized in a deductive system which begins with metaphysics and ends in moral philosophy... is an idea common to both Descartes and Spinoza.... Call it the ideal of the unity of science (E. Curley, *Behind the Geometrical Method* 1988: 5–6)

My purpose in this dissertation has been to understand the "ideal of the unity of science" Edwin Curley ascribes to Descartes and Spinoza, an ideal that was supposedly common to many early-modern philosophers. As Curley points out, that ideal was primarily inspired by geometrical demonstrations: Just as propositions were ordered in a geometrical demonstration, the early-moderns thought, so should propositions be ordered in an overall system of knowledge. Science, the argument went, had to proceed *more geometrico*.

But does this ideal fit early-modern philosophical practice? Apart from Spinoza, who emulated the Euclidean style (but perhaps was less successful in emulating Euclidean rigor), one is hard-pressed to find clear-cut examples. No doubt, it is easy to find early-moderns who placed a premium on certainty, objectivity, clarity of expression, and accessibility from a first-person perspective. However, it is much harder to find a philosopher who actually constructed a philosophy that was organized as a true deductive system. If we read the demand for deductive system-construction strictly, we must admit (or so it seems to me) that early-modern philosophers miserably failed in achieving, and for the most part in even nearing, the ideal they had set for themselves.

But must we be strict? In the foregoing chapters, I've to tried to put in place the resources for expanding, in a particular direction, what we might understand by "science *more geometrico*". That expansion rests on the historical fact that by "geometry" early-moderns understood something different than what we normally understand by the term.

Precisely, they understood geometry to include some of those sciences that we nowadays label 'the applied mathematical sciences'. These 'mixed-mathematical', 'subalternate' or 'subordinate' sciences were taken as *branches* of geometry; that is, as sciences that are inextricably grounded to their mother discipline, but not entirely reducible to it. In the first part of this dissertation (Chapter 2 and 3), I discussed the extant philosophical thinking about these disciplines, focusing on the concepts of "tempered-hylomorphism" (a neologism) and "subalternation" (the real deal). I used these concepts to explicate the ways in which the mixed-mathematical sciences were grounded in and irreducible to their mother discipline. In Chapter 3, I also showed that in early-modern scholastic-Aristotelianism, the notion of "subalternation" was removed from its original domain of appliation and used to analyze the relationship of *non*-mathematical disciplines, particularly the relationship of metaphysics to the other speculative sciences. The crucial point here is that although this set of ideas is not 'geometrical' in any strict sense, it was taken from reflection on the nature of geometry and its branches. Thus, I'd like to argue, when early-moderns looked onto geometry as an exemplar worth emulating in their new sciences, they saw not only its certainty, clarity, objectivity, etc., but also that set of concepts studied in this dissertation. This was neither revolutionary nor exceptional on their part—those concepts were simply part of a standard machinery through which geometry and its offshoots were understood. Especially within scholastic-Aristotelianism, this would have been par for the course. Thus, I argue, when it came to constructing science *more geometrico*, early-moderns would have been as likely to construct a science that straddled disciplinary boundaries and 'borrowed' principles from one domain and applied them in another as they were to construct a science that mimicked Euclid's *Elements* directly. In order to provide support for this thesis, in the second part of this dissertation (Chapters 4 and 5) I argued that two of the seventeenth-century's new sciences can be fruitfully analyzed by the conceptual resources set out in the first part. I shied away from attributing intent to the authors of the *Discorsi* and *Principles* simply because I do not believe my case is strong enough. Nevertheless, I do believe that an early-modern who might have approached these scientific texts expecting to find those concepts studied in Part I (say, an average scholastic-Aristotelian or even someone who was only educated in that tradition) would have not been disappointed. I take this to be at least a plausibility argument to the effect that a wider interpretation of "science *more geometrico*" is warranted. If such an interpretation is warranted, however, some early-modern philosopehrs (notably Descartes in the *Principles*) were much closer to achieving the ideal of the unity of science than we have thus far realized.

APPENDIX A

EUSTACIUS A SANCTO PAOLO: *SUMMA PHILOSOPHIAE QUADRIPARTITA*

The following is the fifth *quaestio* in Book I (*Logica seu Dialectica*), part III (*de ijs quae spectant ad tertiam mentis operationem*), tract III (*de tribus praecipuis Syllogismi speciebus*), first disputation (*de syllogismo demonstrativo*) of the *Summa*, p. 235 of the 1609 edition. Marginal notes in the original are rendered in the footnotes.

> *De Scientia posteriori modo sumpta, quae est una per aggregationem.* Scientia hoc posteriori modo sumpta non solius demostrationis, sed etiam divisionis, definitionis, inductionis , ac experientie effectus dici potest, nec aliter una dici potest, quam exercitus, respublica, coetus hominum unus dicitur: neque adhuc solis conclusionibus demostratis, sed etiam rebus definitis, partibus divisis, principiis quoque inductione & experientia probatis constat. De[1] hac igitur sicut & de priori scientia non nullae difficultates sigillatim explicandae sunt. Primum quaeritur, qualenam esse debeat obiectum totalis eiusmodi scientiae. Respondetur[2] has esse conditiones praedicti obiecti. Prima,[3] ut sit id cuius gratia cetera cognoscuntur, id est, ut sit id ad cuius pleniorem notitiam caetera quae in illa scientia traduntur, pertineans. Secunda,[4] ut sit universale quid, est enim rerum universalium scientia. Tertia,[5] ut sit quid unum, idque vel synonymum , vel analo gum. Quarta,[6] [236]
> ut sit ens verum, & reale. Quinta,[7] ut sit necessarium, vel certe contingens, ut plurimum. Sexta,[8] ut sit adaequatum, ita ut nec magis, nec minus late pateat quam scientia. Septima,[9] ut duo in eo considenti possint; Alterum instar materiae, videlicet res ipsa quae

[1] *Prima difficultas, de Obiecto totalis scientia.*
[2] *Conditiones praedicta obiecti.*
[3] *Prima.*
[4] *Secunda.*
[5] *Tertia.*
[6] *Quarta.*
[7] *Quinta*
[8] *Sexta*
[9] *Septima*

consideranda est; alterum instar formae, videlicet ratio sub qua res eiusmodi consideranda est in tali scientia. Octana,[10] ut sit mensura scientiae quoad unitatem , distinctionem , nobilitatem, veritatem , certitudinem , evidentiam , ordinem, & amplitudinem. Nona,[11] ut habeat principia, partes & proprietates quae de ipso generatim deque eiusdem partibus speciatim per principia demonstrentur.

Secundo[12] quaeritur undenam petatur unitas & distinctio eiusmodi totalium scientiarum, verbi gratia, unde habeat Physica quod sit ac dicatur una scientia ab aliis distincta. Respondetur id petendum esse ab obiecto totali, adaequato, ita ut sit una totalis scientia cuius unum est genere totale obiectum in ratione obiecti, ad quod omnia quae in ea scientia traduntur, referri possunt: Sic Dialectica dicitur una totalis scientia, quia tota versatur circa unum genus obiectu totale, nempe circa actionem mentis dirigibilem iuxta optimum cognoscendi modum , quod quidem in ratione obiecti differt a quovis alterius scientiae obiecto, unde & Dialectica differt a quavis alia totali scientia , idemque de morali Philosofia dicendum , quae versatur circa actionem moralem seu humanam, ac liberam quatenus ad optimum vitae finem dirigibilis est. Quod etiam spectat ad tria illa genera theoreticarum [237]
scientiarum nempe Physicam, Mathematicam, & Metaphysicam, distingui solent penes triplicem[13] obiecti abstractionem à triplici materia, ut docet Aristot. Quod ut planius intelligas, sciendum est triplicem esse materiam à qua postest intellectus aliquod objectum abstrahere, nempe materiam singularem seu conditiones individuantes; sensibilem seu accidentia sensu perceptibilia; denique intelligibilem, seu quantitatem, quae per se sola non cadit sub sensum: unde ortum triplex[14] genus abstractionis, nempe à materia singulari, à materia sensibili, & à materia intelligibili, proindeque triplex genus theoreticarum scientiarum; nempe[15] Physica, cuius objectum nempe end naturale abstrahitur à materia singulari duntaxat, non item a materia sensibili, aut intelligibili.

Nam Physicus[16] considerat quidem entia naturalia in univetsum abstracta à singularibus, non vero abstracta à qualitate sensibili, aut quanitate; deinde Mathematica, cuius obiectum, nempe ens Mathematicum,[17] ab omni fermè materia sensibili non tantùm à singulari abstractum est: Mathematicus enim considerat quantitatem vel puram, id est, omni qualitate sensibili distitutam, ut in Arithmetica & Geometria; vel cum una aut altera duntaxat specie qualitatis coniunctam, ut in musica, quanitatem cum sono, in Astrologia quantitatem cum motu coelesti, in Optica quantitatem cum lumine vel coloe; denique Metaphysica, cuius obiectum nempe ens, ut ens,[18] abstrahitur ab omni materia non tantùm singulari aut sensibili, sed etiam ab intelligibili, ut patet. Ubi aduerte,[19] quoniam in mathematicis non unus [238]
plane est abstractionis modus a materia sensibili, ideo penes diversum illum abstractionis modum diversa in illis esse totalium scientiarum genera, quod non contingit in physicis, aut metaphysicis. Sed haec satis de distinctione totalium scientiarum.

Tertiò quaeritur de ordine scientiarum inter se, quaenam sint priores, puae posteriores.[20]

[10] *Octana.*
[11] *Nona.*
[12] *Secunda difficultas, unde petatus Unitas & distinctio totaliu scietiarum*
[13] *Triplex materia à qua obiectum intellectum potest abstrahi.*
[14] *Triplex abstractio in scieniiis [??]*
[15] *Scientia theoretica triplex.*
[16] *I Physica*
[17] *Secunda Mathematica.*
[18] *3. Metaphysica.*
[19] *Nota.*
[20] *Tertia difficultas. De ordine scientiarum. Ordo quadruplex.*

Et quoniam ordo quadruplex est paecipuè, nempe ordo naturae, dignitatis, doctrinae, temporis, primùm comparandae sung scientiae practicae com theoreticis, deinde theoreticae ad inuicem conserendae erunt iuxta illum quadruplicem ordinem.

Ordine igitur naturae, theoreticae practicas antecedunt.[21] Nam verum simpliciter est natura prius vero practico. Item ordine dignitatis theoreticae practicis anteponendae sunt, eoquod earum finis, quae est contemplatio, praestantior sit istarum fine, quae est actio: adde quod illae ad divinorum intuitum nos excitant, hae verò ad res humanas nos deprimunt: at verò ordine doctrinae practicae praecedunt theoreticas, eoquòd animum praeparant ad faciliorem naturae rerumque divinarum contemplationem, licèt quatenus à quibusdam scientiis theoreticis sua principia desumant, sint aliqua ratione etiam secundum ordinem doctrinae illis posteriores; item ordine temporis practicae theoreticas antecedunt. Nam inventis, inquit Aristotel. I. Met.c.I, iam & institutis iis scientiis, quae ad necessitatem aut voluptatem vitae omnium pertineban cuiusmodi sunt scientiae practicae, ac praesertim artes quaedam, [239]
postea excogitatae sunt scientiae speculativae quae nec ad sensuum voluptatem, nec ad naturae necessitatem pertinebant.

Quod spectat ad scientias theoreticas, Physicam Methmaticam, & Metaphysicam.[22] Ordine naturae Metaphysica antecedit, sequitur Physica, tandem Mathematicae, eoquod obiectum Metaphysicae natura est omnium primum ut pote universalissimum; obiectum verò Physicae natura etiam prius est obiecto Mathematicarum, quia est communius: Adde Mathematicas esse inferiores, & subalternas Physicae, ut pote à qua sua principia ut plurimum desumunt:

Item ordine dignitatis Metaphysica praeit,[23] quia res natura praestantissimas, nempe spirituales substantias & communia rerum omnium principia contemplatur: sequitur Physica quae antecellit Mathematicas tùm ratione subiecti. Illa enim substantias; haec verò tantum accidentia contemplatur, tùm rerum quas considerat varietate, sicque ultimatum locum tenent Mathematicae. Ordine doctrinae seu facilitatis praecedit reliquas Physica,[24] si viam inventionis spectes. Prius enim & facilius rerum Physicarum principia, causae & rationes inveniri possunt, quàm rerum Mathematicarum quae à sens[e] sunt remotiores: si verò viam disciplinae consideres, Mathematicae Physicam antecedunt, facilius enim & ordinatius docentur simul & addiscuntur, sicque ordine facilitatis ultimum locum tenet Metaphysica.
Denique ordine temporis[25] probabilissimum est Physicam antecessisse Mathematica. Primum enim omnium naturalism rerum principia causas priprias affectiones, * effectus rimati sunt homines, deinde caolorum & siderum motibus excitati, sese ad Mathematicas disciplinas, imprimisque ad Astrologiam applicuerunt; tum ex his inferioribus scientiis ad Metaphysicam seu primam Philosophiam se contulerunt. [240]

Quarto & ultimo quaeritur de subalternatione scientiarum, quaenam scientiae aliis sint subalternae et subalternatae.[26] Ut autem appellant subalternatam, inferiorem vero subalternantem, impropriam eorum locutionem praetermittam qui superiorem scientiam dicitur in genere scientia alteri subalternata que illi subest seu subiicitur, atque ab ea dependet:[27] illa autem subiectio seu dependentia dicitur subalternatio.[28] Et quia una scientia ab altera

[21] *Quo ordine theoreticae sunt priores practicis & è contra.*

[22] Quanam sit ratio prioritatis & posterioritatis inter theoreticas, Ordine natura.

[23] Ordine dignitatis.

[24] Ordine doctrina

[25] Ordine temporis.

[26] Quarta difficultas, de subalternatione scientiarum.

[27] Quid sit scientia subalternata.

[28] This rather ungrammatical sentence is corrected in the 1648 Cambridge edition of the *Summa* to read:

pendere potest vel secundum aliquam sui partem duntaxat, vel secundum se totam, sive ab aliqua parte scientiae superioris, sive a tota illa scientiae pendeat, hinc subalternatio in genere duplex:[29] una secundum partem quae est minus propria, altera secundu totam, quae est magis propria & de qua praefertim hic agitur.[30] Contingit autem haec subalternatio triplici ratione, primum quidem ratione finis, cum nempe finis inferioris scientiae refertur ad finem superioris. sic finis Dialecticae,[31] qui est mentis operationes ad legitimum cognoscendi modum dirigere, refertur ad finem theoreticarum scientiarum nempe veritatis contemplationem, ideoque Dialectica caeteris scientiis tanquam administra subalternatur.

Secundo[32] subalternatio contingit ratione subiecti seu obiecti, cum nempe subiectum seu obiectum unius subalternis obiecte continetur. sic Astrologia subalternatur ei parti Physi- [241]
ologiae, quaede coelo disserit; Physici enim est in ea parte non tanum de substantia coelesti, sed etiam de ipsius quantitate & motu dissere, que etiam ex instituto contemplatur Astrologus. Tertio[33] denique subalternatio contingit raione principiorum, cum nempe principia inferioris scientiae a superiori scientiae desumuntur, itaut principia illa inferioris, sint conclusiones superioris, sin enim quaedam sunt principia Geometrica, verbi causa, quantitatem continuam esse in infinitum dividuam, a quovis puncto ad quodvis punctum duci posse lineam, quae desumuntur ab ea parte Physicae, quae de quantitate, quatenus est proprietas corporis naturalis ex instituto disserit. Ibi enim demontratur ee propositiones quae tanquam verae supponuntur in Geometria.

Notem[??]dum[34] autem est hoc loco, licet una eadem scientia possit alteri subalternari triplici illa consideratione, quomodo Dialectica subalternatur ei parti physice que est de anima, nihilominus in vera & legitima subalternatione, non tam haberi rationem finis, quam subiecti & principiorum, itaut tunc legitima cemfeatur subalternatio, cum una alteri ratione utriusq, nempe subiecti simul & principiorum subalternatur. Natamdum[35] etiam illud est, nempe subiectum inferioris, seu subalternatae scientiae ita debere contineri sub obiecto alterius, ut ab ipso differat penes differentiam aliquam accidentariam non essentialem, quae quidem sit ratio formalis, sub qua tale obiectum consideratur in tali scientia. Sic enim obiectum Opticae seu Perspectivae differt ab obiecto Geometriae cui subalternatur. [242]
Geometrica enim considerat lineam quantenus linea est, at perspectiva considerat eamdem quantenus visualis est, seu quantenus ad visionem potest pertinere, esse autem visuale, est aliquid plane accidentarium ad rationem Linae.

Ex iis[36] autem facile est intelligere scientias practicas subalternari theoreticis, deinde Mathematicam subalternari Physicae; immo inter Mathematicas eas quae dicuntur impurae, sue quae considerant quantitatem cum aliqua specie qualitatis, nempe Musicam, Astrologiam, Opticam, subalternari puris Mathematicis, Arithmeticae nempe vel Geometriae; Denique

Ut autem improptiam eorum locutionem pretermittam qui superiorem scientiam appellant subalternatam, inferiorem vero subalternantem; dicitur in genere scientia alteri subalternata, quae illi subest seu subjicitur, atque ab ea dependent: illa autem subjectio seu dependentia dicitur subalternatio.

[29]Subalternatio in genere triplex.
[30]Subalternation propria duplex.
[31]Prima retione finis.
[32]Secunda ratione objecti.
[33]Tertio ratione principiorum.
[34]Nota:
[35]Nota.
[36]Corolarius, Quaenam scientia aliis scientiis subalternentur.

omnes scientiae Metaphysics,[37] licet non omnino proprie aliqua tamen ratione subalternari, tum quia Metaphysica continet generalissima quaedam principia, quibus pricncipia caeterarum scientiarum saltem ductu ad incommodum confirmari possunt; tum quia subiectum Metaphysicae; quod est ens in suo conceptu, saltem confuse caeterarum omnium scientiarum subiecta continet.

[37]Omnes scientia subalternatur Metaphysica.

APPENDIX B

FRANCISCO SUÁREZ:*DISPUTATIONES METAPHYSICAE*

The following is the concluding segment of Section V (*Utrum metaphysica sit perfectissima scientia speculativa, veraque sapientia*) of the first metaphysical disputation (*De natura primae philosophiae seu metaphysicae*).

Expeditur dubium de subalternatione aliarum scientiarum ad metaphysicam

46. Aliquorum opinio.— Hic vero dubium occurrit circa conditionem hanc, an hoc imperium vel directio metaphysicae in alias scientias tale sit, ut dicendae sint omnes scientiae ratione illius subalternari metaphysicae. Non enim defuere qui ita de hoc imperio metaphysicae existimaverint. ut ratione illius omnes scientias subalternatas, solam vero metaphysicam simpliciter, aut tantum subalternantem esse dixerint. Quam sententiam aliqui tribuunt Aristoteli, I Phys., c. 2, et lib. I Poster, c. 7; et Platoni, lib. VII de Repub., ubi de metaphysica nomine dialecticae disputat. Eamque insinuat D. Thomas, opusculo de natura generis, c. 14. Alii vero simpliciter negant hanc subalternationem; et haec est communis et recepta sententia, ut videre licet in Iavello, I Metaph., q. 2; Soncinate, lib. IV, q. 9; Soto, I Phys., q. 11. Alii denique distinctione utuntur, et sub una ratione seu usu vocis subalternationis docent posse metaphysicam dici subalternantem, saltem aliquo modo, simpliciter vero negant. Lege Fonsecam, lib. IV, c. 1, q. 1.

47. Quae sit subalternata scientia.— Ne vero in nominum ambiguitate versemur, supponamus proprie scientiam illam dici subalternatam alteri, quae essentialiter seu necessario ex natura rei ab illa pendet in esse scientiae, ita ut esse scientia non possit nisi scientiae subalternanti coniungatur, et ab illa sumat evidentiam principiorum. Ratio autem huius est, quia scientia subalternata non habet principia per se nota et immediata, sed conclusiones demonstrabiles in superiori scientia; et ideo, sicut omnis scientia ab habitu principiorum essentialiter pendet, ita subalternata a subalternante proprie dicta, quia utraque sumit a superiori virtute evidentiam principiorum. Dices: unde constat subalternatam scientiam non posse habere principia per se nota, sed conclusiones, alibi demonstrabiles, illi esse principia? Respondetur, hoc non posse nisi ad significationem vocis pertinere; nam in re constat

esse aliquas scientias quae huiusmodi utuntur principiis, ut medicina, musica, etc.; has ergo dicimus subalternatarum nomine significari. Nam illae quae principia habent immediata, proxime ac per se subordinantur habitui principiorum; et ideo non est cur respectu alterius scientiae subordinationem habere dicantur, cum ab illa per se non pendeant; haec ergo dependentia unius scientiae ab alia nomine subalternationis significatur.

48. Quae conditiones ad subalternationem requisitae.— Ex quo fit, ut subalternatio vera non sit nisi inter scientias diversas; nam etsi in eadem scientia sit dependentia unius conclusionis ab altera usque ad principia, non tamen ideo scientia dici potest vel in totum, vel in partem subalternata, cum absolute tota scientia non alteri scientiae priori, sed immediate habitui principiorum subordinetur, sed ad summum dici potest una conclusio subordinata vel subalternata demonstrationi alterius. Oportet ergo ut scientiae distinctae sint, et quod inter se habeant praedictam dependentiam et subordinationem. Contingit vero interdum scientiam aliquam non in omnibus suis principiis, neque in omnium conclusionum demonstrationibus, sed in quibusdam habere dictam dependentiam a scientia superiori; et tunc dicitur illi subalternata, non in totum, sed ex parte, seu partiali subordinatione, non totali; quo modo geometria dicitur subalternari naturali philosophiae, quia, licet utatur multis principiis indemonstrabilibus, aliqua tamen habet quae in philosophia demonstrantur, ut illud: A quolibet puncto in quodlibet punctum lineam duci, quod in physica demonstratur, quia nulla indivisibilia sunt immediata, eo quod ex indivisibilibus componi non possit continua quantitas.

49. Unde enascatur scientiae unius ad alteram subalternatio.— Oriri autem solet haec dependentia unius scientiae ab alia ex subordinatione obiectorum; nam, sicut esse scientiae consistit in ordine ad obiectum, ita et principia sunt proportionata illi. Quapropter si obiecta duarum scientiarum non sunt inter se subordinata, utpote si sint genera vel species omnino condivisae, inter illas scientias non potest esse subalternatio. Oportet ergo ut haec subalternatio in obiectis fundetur, nimirum in eo quod obiectum unius est idem cum obiecto alterius, adiuncta aliqua differentia accidentali, quae in esse entis sit per accidens, in esse autem scibilis sit aliquo modo per se, et constituat speciale obiectum scibile. Quando enim duo obiecta scientiae per se subordinantur, etiam in esse rei, scilicet ut genus et species, vel ut superius et inferius essentialiter, scientiae de illis obiectis non possunt esse subalternatae, saltem totaliter, quia, vel pertinent ad eamdem scientiam, si in eadem omnino sint abstractione, vel certe si scientiae diversae sint, erit utraque subalternans, quia utraque habere potest propria principia sumpta ex propria differentia obiecti quod considerat, vel ex prima passione, et per illa poterit demonstrare reliquas conclusiones quae de posterioribus passionibus conficiuntur. Nam scientia de homine non considerat quae conveniunt homini ut animal est, sed tantum ut rationalis est, et in illis non subalternatur scientiae de animali, quia esse rationale immediate convenit homini, et ex hoc principio oriuntur aliae passiones hominis ut homo est. Quod si aliqua est quae pendeat ex gradu sensitivo ut sic, aliquo modo, aut ex speciali coniunctione sensibilis cum rationali, quantum ad id subalternabitur partialiter scientia de homine scientiae de animali, non tamen omnino et totaliter.

50. Ad subalternationem ergo absolutam et totalem, necesse est ut subiectum subalternatae scientiae addat accidentalem differentiam subiecto scientiae subalternantis, ut linea visualis addit lineae, numerus sonorus numero, humanum corpus sanabile humano corpori; nam ex hac coniunctione provenit, tum ut scientia quae specialiter considerat proprietates ex illo coniuncto ut sic manantes, sit diversa a scientia quae abstrahit ab illa compositione et subiectum secundum se considerat, tum etiam ut principia talis scientiae sint conclusiones superioris scientiae, quia nimirum proprietates talis compositi oriuntur ex ipsis componen-

tibus, et ex proprietatibus quas secundum se habent, et in superiori scientia demonstrantur. De quibus omnibus in lib. I Post., c. 11, latius disseritur; hic vero solum insinuata et quasi delibata sunt, ut breviter explicemus quo modo metaphysica sit affecta ad alias scientias, etiam quoad hanc proprietatem.

51. Scientiae subalternantis proprietates nullae insunt metaphysicae.— Ex dictis igitur non obscure colligitur nullam proprietatem scientiae proprie subalternantis convenire metaphysicae respectu aliarum scientiarum. Primum enim non pendent omnino aliae scientiae in esse scientiae a metaphysica, quia non pendent in omni evidentia et certitudine suorum principiorum. Habent enim sua principia immediata, et indemonstrabilia ostensive et directe; quod satis est ut possint habere evidentiam eorum immediate ab habitu principiorum, quae sufficit ad generandam scientiam. Etenim, licet metaphysica demonstrare possit aliquo modo illa principia, illa tamen demonstratio non est simpliciter necessaria ad iudicium evidens talium principiorum, cum ex terminis possint evidenter cognosci, et illa demonstratio non sit proprie a priori, sed per deductionem ad impossibile, vel ad summum per aliquam extrinsecam causam. Metaphysica ergo non est simpliciter necessaria ad evidentiam horum principiorum; ergo neque ut habitus ex eis genitus sit vera scientia; ergo talis habitus non est scientia subalternata metaphysicae. Rursus obiecta inferiorum scientiarum non sunt per accidens subordinata enti aut substantiae; sed per se et essentialiter, ut patet in ente naturali, quod est obiectum philosophiae, et de quantitate, quae est obiectum mathematicae. Et ratio est, quia sub ente nihil continetur per accidens, sed per se; quod si sit aliqua scientia quae agit de aliquo ente rationis, illa nullo modo subordinatur metaphysicae quatenus agit de ente reali, quia ens rationis ut sic non continetur sub ente reali, sed est primo diversum; quatenus vero metaphysica agit de ente rationis, sic quodlibet ens rationis non per accidens sed per se continetur sub ente rationis ut sic, quod metaphysicus considerat; non ergo intercedit propria et totalis subalternatio. Quod ipse etiam usus docet, alias esset metaphysica ante omnes scientias acquirenda, quoniam sine illa nulla scientia esse posset; oppositum autem habet usus propter causas supra tactas, et nihilominus verae demonstrationes fiunt ex principiis per se notis sine metaphysica, et praesertim in mathematicis; non intercedit ergo vera subalternatio. Quod autem in una vel altera conclusione intercedat interdum, id non repugnat, ut ex principiis positis facile intelligi potest.

52. Quod si extenso vocabulo quis velit subalternationem vocare illam excellentiam, et quasi imperium, quod metaphysica habet in alias scientias, quatenus earum principia potest aliquo modo stabilire et confirmare, et quatenus magnam lucem omnibus affert, vel quatenus attingit ultimum finem vel felicitatem hominis, non est cum eo contendendum, cum de nomine disceptatio sit, praesertim cum graves auctores interdum eo genere locutionis utantur, ut videre licet in Simplicio, lib. I Phys., text. 8; et Themist., in Paraph. ad I lib. Poster., c. 2; Aristoteles vero nunquam est usus illo loquendi modo, neque illam proprietatem ad rationem sapientiae requisivit, sed solum ut aliis scientiis quodammodo dominetur, quod longe diversum est, ut ex dictis constat.

53. Interrogationi respondetur.— Hactenus ergo satis probata est secunda assertio, metaphysicam, scilicet, esse veram sapientiam. Interrogabit vero aliquis quo modo prior et posterior assertio cohaereant; nam Aristoteles in Ethicis scientiam et sapientiam ponit species condistinctas sub genere virtutis intellectualis; nos vero doctrinam hanc simul facimus scientiam et sapientiam. Hoc autem facili negotio expeditur, si dicamus cum D. Thoma, I-II, q. 57, a. 2, ad 1, sapientiam condistingui a scientia, non quia scientia non sit, sed quia in latitudine scientiae habeat specialem gradum et dignitatem. Unde fit scientiam dupliciter sumi: uno modo generice, ut dicit habitum per demonstrationem acquisitum, ut in I Poster.,

c. 2, definitur, ubi hac ratione nulla mentio fit sapientiae in particulari, quia solum agitur de scientia sub illa generali ratione, sub qua sapientiam complectitur, et sic procedit prior assertio a nobis posita. Alio modo accipitur scientia magis stricte, prout dicit habitum, qui solum versatur circa conclusiones demonstrabiles, et non circa ipsa principia, id est, pro habitu, qui solum scientia est, et nullo modo intellectus, ad eum utique sensum quo Aristoteles dixit sapientiam esse intellectum et scientiam, et in hoc sensu scientia distinguitur a sapientia, et sic dicimus metaphysicam non esse scientiam huiusmodi, sed sapientiam.

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