

HOBBS ON THE ORDER OF SCIENCES: A PARTIAL DEFENSE OF THE MATHEMATIZATION THESIS

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ABSTRACT: Accounts of Hobbes's "system" of sciences oscillate between two extremes. On one extreme, the system is portrayed as wholly axiomatic-deductive, with statecraft being deduced in an unbroken chain from the principles of logic and first philosophy. On the other, it is portrayed as rife with conceptual cracks and fissures, with Hobbes's statements about its deductive structure amounting to mere window-dressing. This paper argues that a middle way is found by conceiving of Hobbes's *Elements of Philosophy* on the model of a mixed-mathematical science, not the model provided by Euclid's *Elements of Geometry*. I suggest that Hobbes is a test case for understanding early-modern system construction more generally, as inspired by the structure of the applied mathematical sciences. This approach has the additional virtue of bolstering in a novel way the thesis that the transformation of philosophy in the long seventeenth century was indebted to mathematics, a thesis that has come under increasing scrutiny in recent years.

1. INTRODUCTION

The idea that mathematics was central to the transformation of philosophy in the long seventeenth century is well known. It has roots in the work of several twentieth-century polymaths—Alexandre Koyré, E. A. Burttt, and E. J. Dijksterhuis—and has been with us long enough that it is considered a conservative historiographical device.¹ But scholars increasingly dispute its truth. They argue that much early-modern philosophical production was antimathematical both in theory and practice. Mordecai Feingold, for example,

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¹ See Burttt 1954, Koyré 1939, Dijksterhuis 1961, and Cohen 1994, chap. 2.

shows that those who accepted the primacy of mathematics in natural philosophy were “a divisive clique that had little in common with the interests and concerns of the generality of membership” of the Royal Society (2001, 78). Roger Ariew, for another example, shows that the varieties of Cartesianism demonstrate that the adoption of mathematical techniques was not central even to those who accepted a good deal of Cartesian metaphysics (2014, chap. 4.3). Finally, Daniel Garber (forthcoming) shows that the concept of a “law of nature”—a decidedly mathematical one from a contemporary perspective—had a history that is largely distinct from the history of mathematization. These are not claims about peripheral countercurrents in the scientific revolution; they are about its core. They thus leave us with a problem: if the use of mathematics was not essential to the transformation of philosophy in the seventeenth century, what becomes of the mathematization thesis?

In this essay, I mount a partial defense of the mathematization thesis. My defense consists of two intertwining claims: first, many discussions of the thesis wrongly characterize, if only by lack of attention, the extension of the term “mathematics” in early modernity; second, this mischaracterization leads us to underappreciate one manner in which mathematics was important for philosophy. I use Hobbes as a test case. My defense is partial because I only outline enough of the Hobbesian background to articulate the main claim; a full defense would require much more. Moreover, I do not intend to resuscitate the idea that mathematization was *the* engine of the scientific revolution. I only wish to show that mathematics played an underappreciated but determinative role in the thought of one philosopher. While I believe that we can find similar instances in works of others, I cannot address them here.² To spell out what is underappreciated in the early-modern use of mathematics, I need to bring up another common element of early-modern philosophy.

It is well known that the project of constructing a “system” of knowledge—a set of theses capable of integrating all that is and could possibly be known—was common to many early-modern philosophers. The most famous are perhaps Hobbes and Spinoza, although they were certainly not the only ones who tried to construct bodies of knowledge that consistently accounted for both experimental and abstract theses. That unity has been often understood in terms of mathematics. Mathematics, the idea goes, provided the organizing principle for philosophical systems, whether or not mathematics also provided the content of those systems. Thus, it was possible for Spinoza to champion the geometrical manner as an organizing principle

² See Biener 2008.

for philosophy and still circumscribe rather heavily the epistemic utility of mathematics.³ In fact, the idea that mathematics provided a model for system construction is almost as prevalent as the mathematization thesis itself. Importantly, however, this idea takes *Euclidean geometry* to provide the relevant structure for studying philosophical systems, even when the philosophical systems studied contained antimathematical theses and criticisms of Euclid himself.⁴ The following represent an orthodox position:

It was Euclid and Archimedes who, fully in the tradition of Plato and Aristotle, established mathematics and the natural sciences as axiomatic-deductive theories: axiomatic in their fundamental principles, and deductive in their logical procedures. Their work has so far been the model for Thomas Hobbes (*Politics*), Baruch de Spinoza (*Ethics*), Leibniz (*Jurisprudence*), William Whiston (*Cosmology*) and many more, and has thus become the unique paradigm for strict science in the Western sense: as science “more geometrico.” (Schönbeck 1994, 184)

What few noticed, however, is that this view of system construction reinforces skepticism about the mathematization thesis! For, if we take the Euclidean model to be the aim of, say, Hobbes and Spinoza, we must also take them to be grand failures. Neither succeeded in creating systems that are actually axiomatic-deductive. Spinoza *presented* his system as axiomatic-deductive, but no student of Spinoza can feign that the claims of Book V of the *Ethics* are deducible in an unbroken chain from the principles of metaphysics.

Hobbes did not present his system in a geometrical fashion, but it is equally hard to find a single line of reasoning that begins with the principles of *De Corpore* and ends in statecraft. Nevertheless, I believe we can rescue him from failure, and we can do so by amending the Euclidean model of system construction. By revising the extension of the term “mathematics,” we can see that what Hobbes, my example in this essay, tried to emulate was different from the strict Euclidean model. Rather, he tried to emulate a more flexible *mixed*-mathematical model, and this model allows us to turn some of his failures into successes. The two claims together establish that there is still room for the mathematization thesis—at least as far as system construction is concerned—if we properly revise

³ The importance of the geometrical manner for Spinoza is a traditional, perhaps dated, position. Recent literature argues that Spinoza thought that very little about reality could be understood through mathematics. See, e.g., Schliesser 2014 and Peterman 2015.

⁴ I mentioned Spinoza above. The same is true for Hobbes, who repeatedly criticized Euclid in *Six Lessons*. Hobbes’s criticism, however, was not leveled at the Euclidean idea of demonstration or the propositional structure of Euclidean geometry. Hobbes was mainly concerned with redefining basic geometrical concepts so that they reveal true causes. He also repeatedly praised Euclid, e.g., in the preface to *De Corpore*. For the causal definitions Hobbes sees as missing from Euclid, see Adams 2014.

what we understand by “mathematization.” I begin by considering how mathematics was used by Hobbes.

2. HOBBES AND GEOMETRY

Hobbes was taken with mathematics, particularly with geometry. Although the story of his Euclidean illumination is apocryphal and self-promulgated, his repeated statements regarding his genuine respect for the field and its importance for human knowledge are numerous and well known.⁵ But what did Hobbes take *from* geometry? What was it about the field that appeared to him so useful? There are several available answers. The most cited holds that Hobbes took from geometry a certain conception of the resolute and compositive methods, a way of moving from consequences to antecedents to consequences; or, given Hobbes’s refusal to segregate, and perhaps confusion between, reasons and causes, a way of moving from effects to causes and back to effects.⁶ A second story is that Hobbes took from geometry a respect for conceptual clarity, a respect for definitions free of vagueness and confusion, and for signs that denote only such definitions. This is usually discussed in the context of Hobbes’s theory of definition, but given the primary role of definitions in Hobbes’s system, its importance extends to his philosophy as a whole.⁷ A third story is that Hobbes took from geometry an insistence on demonstrative intersubjectivity, on styles of proof that could be understood by everyone.⁸ A fourth story concerns certainty, and so on.⁹ I do not wish to dispute any of these, each emphasizes the importance of mathematics in a different way.

But there is also another way Hobbes used geometry: as a model for system construction.¹⁰ This use is understudied, due to a kind of presentism. Much of the secondary literature on Hobbes and geometry takes for granted that the extension of “geometry” is known.¹¹ It assumes that Hobbes

⁵ See Aubrey 1898, 1:332, and Berhardt 1986.

⁶ See, e.g., Sacksteder 1981, 1980, Talaska 1988, Hattab 2014, and Adams 2014.

⁷ See, e.g., Adams 2014 and Pettit 2008, chap. 2.

⁸ See, e.g., Grant 1990 and Machamer 1991.

⁹ See, e.g., Popkin 1992, 9–49, and Tuck 1988.

¹⁰ See also Adams 2014, 2016.

¹¹ Historians of science have not made this error for several decades, but the presentism is still part of work on Hobbes’s philosophy of language and politics, and thus often affects treatments of the unity of his system; see, e.g., Daston and Stolleis 2008, 7, and Deigh 1996. Mostly, the problem is one of omission. By not specifying the extension of “geometry,” contemporary authors invite misinterpretation. This presentism persists despite the fact that questions about the Euclidean structure of Hobbes’s system have long been part of the literature on Hobbes; see, e.g., Schuhmann 1995, although the authorship of *Short Tract*, Schuhmann’s subject, is disputed. Although I ultimately disagree with Schuhmann’s conclusions about the origin of Hobbes’s “structure,” I agree that the source is not straightforwardly Euclidean. I thank Marcus Adams for drawing Deigh’s piece to my attention.

understood by “geometry” simply that body of knowledge explored in Euclid’s *Elements*, supplemented by the work of Apollonius, Pappus, and a few others. Of course, that Hobbes saw Euclid, Apollonius, and Pappus as the pillars of geometry is not up for debate. Nevertheless, things are more complicated.

By the terms “mathematics” and “geometry,” early moderns in the long seventeenth century referred to a wider set of disciplines than what we today understand by “mathematics” and “geometry.” Galileo’s initial university title, for example, was “mathematician,” yet his published work was not a straightforward development of what we find in Euclid, Apollonius, or Pappus. No contemporary objected that he did not deserve his title for that reason. A broad understanding of the term was not unusual. Goclenius, for example, in the 1613 *Lexicon* wrote under the entry *Mathematicae* that not only geometry and arithmetic, but also astronomy, optics, music, and mechanics are accepted as “mathematics.” He then produced a table in which mathematics contains not only the four disciplines just mentioned, but also architecture, geography, and others (see Figure 1).

Descartes, reporting on his youth in the *Regulae ad directionem ingenii*, wrote:

When [I had been led] from the particular study of arithmetic and geometry to a general study of mathematics, I inquired first of all precisely what everyone means by this word, and why not only those two sciences of which we have already spoken, but also music, optics, mechanics, and several others are called parts of mathematics. (CSM I 19; AT X 377)¹²

Finally, Hobbes, in his commentary on White’s *De Mundo* of 1642–43, wrote:

Some parts of philosophy, such as astronomy, mechanics, optics, music, are mathematical; others, still untouched, deal with quantity and number, not merely in theory but with reference to the movement of celestial bodies or that of heavy ones, or to the action of shining or sound-producing bodies; [these] must therefore be counted among the mathematical sciences. (1976)

When early-moderns referred to mathematics, they referred to both the pure and the so-called mixed-mathematical sciences. They referred both to the work of Euclid, Apollonius, and Pappus *and* to the work of Archimedes, Grosseteste, Benedetti, Brunelleschi, Copernicus, Kepler, Galileo, and the large number of practitioners of ballistics, perspective, tuning, navigation, and architecture that sprang up in the renaissance.¹³

¹² Descartes 1984 is abbreviated as CSM, and Descartes 1964 is abbreviated as AT.

¹³ See Machamer 1978 and Dear 1988.

Tabula artium, quæ communi nomine Mathematica dicuntur.

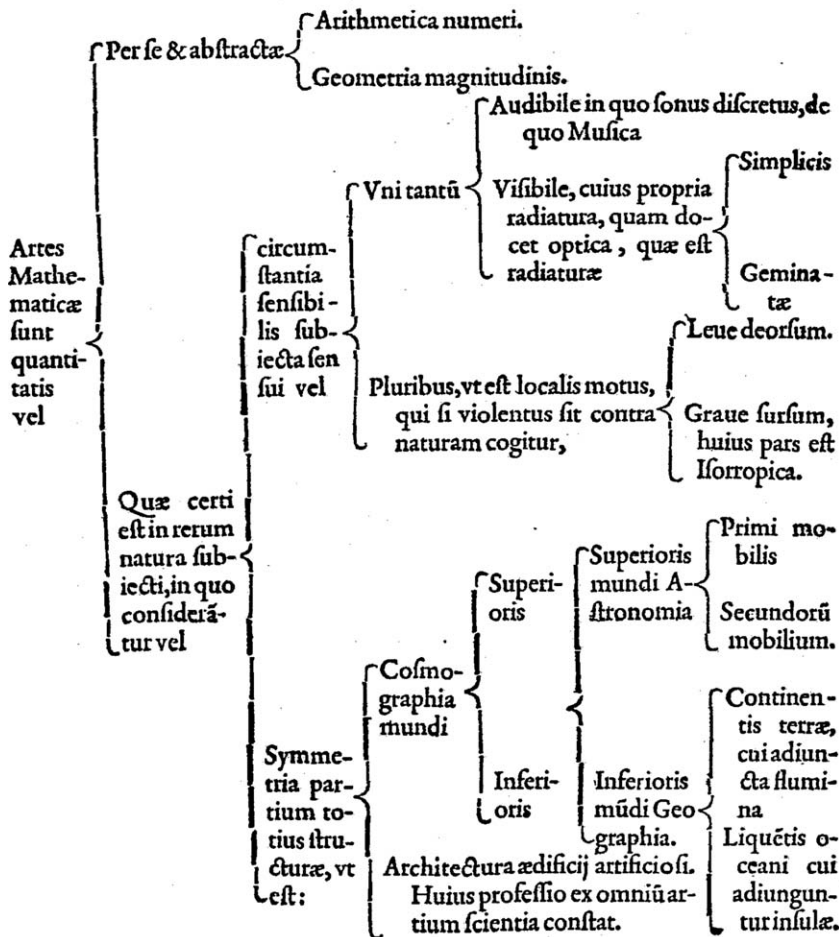


Figure 1: Goclenius, *Lexicon philosophicum* (1613)

We can now return to the initial question, reframed: what did Hobbes take from geometry, if he also took geometry to be constituted by both pure geometry and the variety of mixed-geometrical disciplines? How adding *mixed* geometry to the extension of “geometry” changes the Hobbesian perspective on analysis and synthesis, or the clarity, intersubjectivity, and certainty of mathematics is beyond our scope. In those areas, both pure

geometry and mixed geometry can serve as exemplars, and so teasing out the separate influence of each type is no small feat.¹⁴ However, there is one lesson that Hobbes could have taken *only* from mixed-geometry, one that is unavailable when geometry is considered in its pure form. It concerns the structure of deductive inference, and, given the importance of the deductive model for system-construction, the structure of Hobbes's philosophical system. My claim is that Hobbes took something regarding philosophical system-building from astronomy, optics, mechanics, and music. To see this, a brief overview of the debates regarding the structure of the Hobbesian system is in order.

3. HOBBS'S "SYSTEM"

The debates regarding the structure of the Hobbesian system oscillate between two extremes mapped out decades ago. On the one hand, scholars endorse the demonstrative unity of Hobbes's system. Alan Ryan, for example, echoing earlier work by John Watkins (1965), writes:

Hobbes believes as firmly as one could that behaviour, whether of animate or inanimate matter, was ultimately to be explained in terms of particulate motion: the laws governing the motions of discrete material particules were the ultimate laws of nature, and in this sense psychology must be rooted in physiology and physiology in physics, while the social sciences, especially the terminology of statecraft, must be rooted in psychology. (1970, 102–3)

Such statements are easy to make, but explicitly tracing science to its roots is another matter. Ryan's focus on (some of) Hobbes's pronouncements does not easily match Hobbes's actual practice. One is hard-pressed to find the connection between, say, Hobbes's statecraft and physics, spelled out in his work. It is precisely such a mismatch that leads to the other extreme of Hobbesian scholarship. Tom Sorell holds that:

Hobbes was a firm believer in the unity of science, but he did not think that deductive links kept the body of science whole. . . . That is, he did not suppose that the sciences could all be ranged in a series that the mind could take in as it takes in a geometrical demonstration. (1988a, 520)

The problem is there is evidence for both interpretive extremes in Hobbes, and this is precisely why this debate has gone unsettled. Noel Malcolm provides a clear description of the difficulty:

¹⁴ Adams (2014) admirably teases out the separate influences insofar as certainty and clarity are concerned.

If we attempted to follow Hobbes' "method" through, ascending from one level of knowledge to the next, we would find that each new level required the introduction of concepts which were simply not contained in the subject-matter of the previous level. [For example,] physics will give us the concept of "motion toward" and "motion away from"; but only psychology will provide the concept of "desire" and "fear." (2002, 147)¹⁵

Nevertheless, there have also been several attempts to bridge extremes. An appealing, but relatively ignored, proposal is due to Peter Machamer and Spyros Sakellariadis (henceforth, M&S).

In an essay on "The Unity of Hobbes Philosophy," M&S (1989) respond to the above interpretive dilemma by invoking Hobbes's nominalism. Their argument is based on the fact that for Hobbes the fundamental linguistic unit is the *name* and that "a universal [name] gains its meaning from the various particular objects it supports" (31). Hobbes may sometimes speak as if a single name can be applied to various objects because of "their similitude in some quality or other accident," but, as M&S stress, this similitude is a consequence of *defining* the name under whose extension these objects fall. Defining a name specifies how objects in its extension are to be conceived, and so understanding of their similitude "in some quality or other accident" depends on the definition of the quality or accident in question. It follows from this view that a name can apply across domains which may appear essentially different to a nonnominalist, and by so doing *unify* those domains.¹⁶ Moreover, no domain is privileged. The name does not apply *strictly* in one domain but only *loosely* or *metaphorically* in another. Rather, the name captures all objects in its extension equally. M&S write:

The effects of this nominalism are readily seen in the case of the concept of motion. "Motion" is a universal term and therefore as our knowledge of it "we have in the first place (its) definition" (*De Corpore*, 70). Thus, through its application in different realms we come to know about motion. For example, in the case of natural philosophy, motion is the endeavor or motion of corporeal objects, defined as the continual relinquishing and acquiring of places of the objects. In moral philosophy, the motions are the endeavors or motions of the mind, namely "appetite, aversion, love, benevolence, hope, fear, anger, emulation, envy, etc." (*De Corpore*, 71). The fundamental law of motion, for example, refers both to the fact that corporeal objects continue to endeavor to change their places unless hindered, and to the fact that human beings

¹⁵ Malcolm's view is consonant with the mixed-mathematical view. In fact, I take the mixed-mathematical view defended in this paper to provide a concrete model by which to articulate the introduction of new concepts at different levels of Hobbes's system. I thank an anonymous referee for urging me to clarify this point.

¹⁶ See also *Leviathan* §4.6. Pettit (2008, chap. 2) explores problems with this notion of similitude.

continue to have appetites and aversions (both endeavors), and, unless hindered, will change their place in order to accomplish the goals of those motions. (1989, 31–32)¹⁷

The view is attractive, but not without its problems. First, M&S seem to assert both that the meaning of Hobbes's key terms is univocal across domains and that it (partially) varies. For example, they contend that “universal terms, like ‘body’, ‘law’, ‘motion’, etc., . . . are used *with the same meaning* throughout Hobbes's system” (1989, 20, 32). But they also suggest that “the meaning of the term ‘body’ employed in any particular realm (e.g., politics) is determined not only by that realm itself, but also by the other realms in which it applies (e.g., natural philosophy)” (1989, 31). The two are at odds. The second suggestion implies that the term “body” is *not* used with the same meaning in the physical, moral, and political levels. Rather, it has a wide, hybrid meaning that is narrowed when any domain is specified. Similarly, according to the first suggestion, if a term had the *same* meaning everywhere, there would be no reason to consult *other* domains when trying to understand its meaning in a given domain. Conceiving of its extension in one domain would be sufficient. Its meaning in one realm would *not* be partially determined “by the other realms in which it applies.”

Second, and more importantly, M&S fail to account for the *order* of Hobbes's system. They cannot explain, for example, why one would need to know about the motion of inanimate matter before reasoning about sense. On one hand, if the meanings of terms were *the same* across domains, nothing about the application of a term in one domain would have to be understood for its application in another. One could study, for example, how animal motion has features of “motion” without knowing anything about the motion of inanimate bodies. Knowing about the motion of inanimate bodies would add nothing *at all* to one's knowledge of “motion” in animals.¹⁸ And, if one can reason about domains independently, knowledge of those domains need not be ordered in any way. On the other hand, if the meanings of terms were hybrid—so that “the meaning of [a] term . . . is determined . . . by the other realms in which it applies,” one would need to study *all* realms before fully understanding what a term means in a particular realm. Either way, the order of Hobbes's system gets lost in M&S's account.

¹⁷ Hobbes sometimes presents “scientia” as knowledge of causes and sometimes as knowledge of meanings. The latter allows knowledge of motion to be part of *scientia* even when a causal definition of motion is unavailable. M&S skirt this issue here, and so will I. For an account that stresses that causal reading, see Adams 2014.

¹⁸ Hobbes does allow *some* domains (like politics) to have this kind of independence, but certainly not all. See Sorell 1988b and Malcolm 2002.

Yet there is no context I am aware of in which Hobbes discusses the unity of his system without explicitly highlighting its order. For example, in *De Corpore*, he writes:

In the first place those things be demonstrated, which immediately succeed to universal definitions (in which is contained that part of philosophy which is called *philosophia prima*). Next, those things which may be demonstrated by simple motion (in which geometry consists). After geometry, such things as may be taught, or shewed by manifest action, that is, by thrusting from, or pulling towards. And after these, the motion or mutation of the invisible parts of things, and the doctrine of sense and imaginations, and of the internal passions, especially those of men, in which are comprehended the grounds of civil duties, or civil philosophy; which takes up the last place. And that this method ought to be kept in all sorts of philosophy, is evident from hence, that such things as I have said are to be taught last, cannot be demonstrated, till such as are propounded to be first treated of, be fully understood. (*De Corpore* §6.17)¹⁹

As far as I know, Hobbes is never explicitly concerned with “unity” (or any relevant cognates). Rather, he is concerned with deductive *order*. M&S’s account requires modest modification. This is where the mixed-mathematical model of system construction becomes useful.

4. MIXED MATHEMATICS

Although the term “mixed mathematics” only became common currency in the Renaissance, philosophical reflection on the nature of optics, astronomy, and harmonics traces back to Aristotle’s *Physics* and, more significantly, *Posterior Analytics*. In the latter text, Aristotle holds that scientific demonstrations can furnish true claims regarding a science’s subject matter by showing what properties belong to it by virtue of the kind of thing it is, its genus, not by virtue of any properties that are accidentally true of it. This entails that, for the most part, the premises and conclusions of scientific demonstrations are homogeneous. Thus, metaphysical truths only follow from metaphysical premises, physical truths from physical premises, mathematical truths from mathematical premises, etc.

But Aristotle also admitted of exceptions. In certain sciences, truths regarding one genus could be used in reasoning about another. For example, optics uses geometrical truths to reason about visual rays and harmonics uses arithmetical truths to reason about sounds. In each case, the demonstrations are said to cross-genera: they borrow principles that, strictly speaking, apply only to one kind (namely, continuous or discrete

¹⁹ The above is partially quote by M&S, but they omit the important first sentence.

magnitude) and apply it to another kind (namely, visual ray or sound). Because optics and harmonics borrowed principles from geometry and arithmetic, optics and harmonics were said to “stand beneath the other,” pure mathematical sciences.²⁰

The latinization of Aristotle’s “stand beneath the other” resulted in a technical term that persevered to the early-modern period—“subalternation”—and a body of work built on Aristotle’s relatively few comments on the matter. Robert Grosseteste, in particular, theorized the unique status of the mixed-mathematical or *subalternate* sciences in language that reverberated in many later commentaries, starting with Aquinas and reaching to Suarez and the Coimbrans. According to Grosseteste in his *Commentary on Posterior Analytics*:

A science, is subalternated to another, whose subject adds a condition onto the subject of the subalternating science . . . which condition does not totally derive from the nature of the subject of the subalternating [science], but is assumed from without. (I 18, 41–45)²¹

Consider the science of optics. In geometrical optics, theorems of geometry are used to deduce a broad range of claims about light rays and vision. Why geometry? Because light rays and vision seem to travel in straight lines. If the entities and processes of optics were not partially characterizable by geometrical concepts, geometry would have no foothold. However, the entities and processes of optics cannot be *wholly* characterized by geometrical concepts. If they were, there would be no distinction between geometry and optics. Put differently, a study of *geometrical* optics must use principles that are not essentially geometrical, if geometry and optics are to be different sciences. Here is a common example, due to Richard McKirahan.²² In deducing claims about the apparent size of objects, proofs in geometrical optics use the principle that the larger the angle an object subtends in the visual field, the larger it appears. Given this principle, geometrical optics answers questions like: Given two unequal objects, in what range of positions will the two appear equal? Or, how much smaller will an object appear if viewed obliquely? Equally, geometrical optics can explain why objects can appear equal when they are not, or appear unequal when they are, an explanation that relies on the geometrical configurations of the objects under study. The principle in question, however, is not deducible from the axioms of geometry. Although it uses geometrical concepts (e.g., angle), it is not a geometrical principle. Rather, it is an “added

²⁰ See *Posterior Analytics* I 13. For an introduction to the topic, see McKirahan 1992, Randall 1940, and South 2005.

²¹ See Laird 1983, chap. 1.

²² See McKirahan 1978. The example is from Euclid’s *Optics*.

condition” that concerns the operation of vision in the domain of optics. Combining this principle with the principles of geometry allows us to answer optical questions, but neither the principle itself nor the principles of pure geometry are separately sufficient for the task.

Four main points are relevant here: First, one cannot have a lower, subalternated science without some principles peculiar to its domain. Grosseteste and other Aristotelians offered theories of induction to explain how the principles of subalternated sciences are discovered, but the main point is that they do not come from higher sciences.²³ They are assumed, as Grosseteste put it, from without. Many commentators put the same point in terms of qualities and properties: the lower science must add some quality or property to the higher science, as optics adds “visual” to the line studied by geometry. Principles that characterize the added property—as the above principle characterizes the property of “being seen”—can then be used in demonstrations alongside the principles of the “higher” science.

Second, principles from pure geometry are assumed within optics; they are not justified anew. Many commentators held that an optician is insulated from purely geometrical worries, as a pure geometer is insulated from geometry’s optical application.²⁴ It is the job of the higher science to provide justified principles, but not to apply them, and it is the job of the lower science to apply higher-order principles, but not to justify them. Consequently, no result in optics can cast doubt on results from pure geometry, and proofs in pure geometry are, strictly speaking, beyond the purview of the optician, when she is confined to her “proper business” (Simplicius 1997, 294). We will return to this disciplinary insulation in the conclusion.

Third, in a demonstration that combines a higher and lower science to prove some conclusion in the lower science, the higher is said to provide a deeper explanation of the conclusion. What “deeper” amounts to can vary from commentator to commentator, but, broadly speaking, must answer to one of the Aristotelian causes. Often, the higher sciences were said to provide demonstrations of “the reasoned fact,” “the why,” or simply “the cause,” while the lower sciences provided demonstrations of “the

²³ I bracket a further discussion of the origins of the “extra conditions.” For many working in a broadly Aristotelian mold, the extra conditions were a result of induction, but a review is beyond my scope. The same problem arises for Hobbes, of course, which I also bracket. For the ways in which induction can yield appropriately general, and true, definitions and principles, see Bayer 1997 and Charles 2003.

²⁴ This disciplinary insulation was crucial to the reception of the mixed-mathematical sciences in mid-seventeenth-century Paris. See, e.g., Dear 1988 and Garber 2002.

[unexplained] fact” or “the what.” This broadly causal requirement could appeal to formal, efficient, or even final causes.²⁵

Fourth, going back to our principal question, because optics, due to its subalternation by geometry, is a kind of geometry, an optical demonstration does not just use the basic framework of a geometrical demonstration—it *is* a geometrical demonstration. And, as a *geometrical* demonstration, it comes with all the perks associated with pure geometry: certainty, clarity, intersubjectivity, possible use as part of an analysis/synthesis, etc.²⁶

For Aristotle himself, only the mixed-mathematical sciences (and perhaps medicine) were allowed to cross-genera. However, at the hands of the late scholastics, the concept of subalternation came to have a broad application. By the 17th century, although the *exemplars* of subalternation remained the mixed-mathematical sciences, subalternation was used as a relation that could obtain between any two sciences when one borrows principles from

²⁵ The absence of material causes from the list is due to the fact that the lower science was also said to provide the matter of the demonstration, while the higher science provided the form, or that the lower science considered the objects of the higher science, but enmattered. Either way, appeal to a material cause in the higher science would run afoul of several key characterizations of the subalternating relation. For a single example, take Philoponus’s characterization of the relation of arithmetic and harmonics: “Geometry and arithmetic simply look at double and one-and-a-half and such [relations] separably, while the harmonicist, since he studies such formulas (*logos*) as actually being properties [of something], *cannot even think of them without matter*” (Philoponus 1993, 227, 1 9–13, emphasis added). The various terms intended to capture the relation of causes in subalternating and subalternate sciences take their cue from *Posterior Analytics* I 9, 76a10–16 and are reworkings of Aristotle’s ‘the fact’ (τὸ ὄν) and ‘the reason’ (τὸ διότι).

²⁶ The inference from the “perks” of a subalternating science to the “perks” of a subalternated science is by no means facile. Although I have lumped certainty, clarity, intersubjectivity, and use in analysis/synthesis together, authors may explain the presence of one through the presence of others, and may even explain the presence of all through the presence of more fundamental features like proper appeal to causes. For example, Helen Hattab (2009, 110–19) shows that Ioannis de Guevara, “a priest, friend of Galileo, and Aristotelian,” defended the certainty of mechanics (which he conceived to be subalternate to geometry) first by pressing mechanical demonstration into syllogistic form, and then by using the machinery of Aristotle’s *Posterior Analytics* to argue that mechanical demonstrations make proper use of middle terms, and are thus properly causal. Since they are causal, they can be used to infer causes from effects (*demonstratio quia*, analysis) and effects from causes (*demonstratio propter quid*, synthesis), and thus form an analytic-synthetic chain (the *regressus*). For Guevara, it is ultimately the *regressus*, which guarantees the certainty of mechanical knowledge. I take Hattab’s treatment of Guevara to support the current discussion, by showing how subalternated sciences were thought to inherit not only principles, but also the epistemic characteristics of their subalternating sciences. I thank an anonymous referee for pointing this out. The above inference also relies on the idea that pure geometry itself rises to the level of *scientia*, a notion that was not at all above dispute. In fact, battles regarding the scientific nature of the subalternated science were often part of battles concerning the scientific nature of pure mathematics; on the *Quaestio de Certitudine Mathematicarum (Disciplinarum)*, see Mancuso 1997. For Hobbes’s relationship to the *regressus* and his understanding of the benefits of a “mathematical” method, see also Hattab 2014.

another and applies them in its own, peculiar domain. Subalternation even came to apply to the relationship between metaphysics and the remainder of the sciences. Eustachius a Sancto Paolo, just to take a traditional expository writer, noted:

Amongst Mathematics there are those which are called impure, or which consider quantity with some type of quality; indeed, Music, and Optics are subalternated by pure Mathematics, by Arithmetic or Geometry. And, finally, all sciences are subalternated by Metaphysics. (1609, I 242)

Although opinions differed regarding the extent of the subalternation relation (Francisco Suarez, for example, held that metaphysics does not subalternate all other sciences), it is clear that subalternation was part of the *lingua franca* of late scholasticism. It provided a way to think about inferences that crossed genera, and thus a way to think about broad disciplinary relations.

5. HOBBES'S "SYSTEM" AND MIXED MATHEMATICS

I would like to suggest that one of the things Hobbes took from geometry and his clear knowledge of scholasticism was this distinctly nonreductive, although nevertheless deductive, conception of inference, and with it a nonreductive, but unified, conception of knowledge.²⁷ The approach has the power to span the two interpretive poles mentioned earlier. On the one hand, we should not expect all of Hobbes's sciences to boil down to a few first principles. Taking mixed-geometry as a model does not entail, unlike taking pure-geometry as a model, that all conclusions must boil down to a finite set of axioms about a single, albeit highly abstract, domain. On the other hand, the various fissures between physics and physiology, or physiology and psychology, do not invalidate Hobbes's project as broadly geometrical. Such fissures were part and parcel of mixed-geometry and were caused by the addition of conditions at each transition from a subalternating to a subalternated science. In the quote below, Hobbes considers only "true physics," but his understanding of mixed mathematics is such that any truly mixed science cannot be reduced to the abstract principles of the pure one. Rather, it takes those pure principles and adds ones peculiar to its more specialized domain. Hobbes also expands, as many scholastics and *novatores* before him, the number of the mixed sciences from their paradigmatic instances, to include also physics as a whole:

²⁷ See Leijenhorst 2002 for Hobbes's engagement with late Aristotelianism.

Since one cannot proceed to the consequences or motions without a knowledge of quantity, which is geometry; . . . Therefore physics (I mean true physics), that depends on geometry, is usually numbered among the mixed mathematics [*mathematicas mixtas*] . . . Those mathematics are pure which (like geometry and arithmetic) revolve around quantities in the abstract . . . those mathematics are mixed which in their reasoning also consider any quality of the subject, as is the case with astronomy, music, physics, and the parts of physics *that can vary on account of the variety of species and the parts of the universe*. (*De Homine*, chap. X; OL II 93–94; slightly modified, emphasis added)²⁸

The idea that specific domains are constructed by “adding conditions” to higher, more generic domains can account for the ordering of Hobbes’s system, can retain its unity, and can explain his abiding concern with the example of mixed mathematics. Another famous Hobbesian take on his *Elements* makes the point plain:

Having completed this section, On Man, I have finally fulfilled my promise. For you now possess the prime Elements of my philosophy in all its divisions and subdivisions. Moreover, it happens that the two parts whereof this section consists are very dissimilar. . . . They are therefore somewhat abruptly conjoined; but this was necessary, *granted the method of my work as a whole*.

For man is not just a *natural body*, but also a part of the state, or (as I put it) of the body *politic*; for that reason he had to be considered as both man and citizen, that is, the *first principles of physics had to be conjoined with those of politics* [*ultima physicae cum principiis politicae conjungenda erant*]. . . . (*De Homine*, preface; OL II *v*; emphasis added)

Hobbes’s stated procedure is to take definitions in “higher” domains and add principles specific to lower domains. We also see this in Hobbes’s characterization of the finer transition from Part II of *De Corpore* (“First Grounds of Philosophy,” which include general concepts like velocity and cause) to Part III (“Proportions of Motion and Magnitude,” which include uniform and accelerated motion).²⁹ In brief, the first two sections of Part III are titled: “1. Repetition of some principles of the doctrine of motion set down above” and “2. Other principles added to these” (*De Corpore*, chap. 15; OL I 175). The first section repeats the causal and kinematic principles of motion articulated in Part II, Chapter 8, and the second adds a new concept: endeavor (*conatus*). *Conatus* is defined as “motion made in less space and time than can be given,” and it is the addition of *conatus*, a more highly specified type of motion, to the general principles of geometry that yields the variety of specific motions studied in Part III. For example, “uniform motion” is defined as an equality of impetus—a quantity of *conatus*—at every point of an object’s trajectory (*De Corpore*, 16.1; OL I 184).³⁰ *Conatus* itself is further specified

²⁸ Hobbes 1991 abbreviated as OL.

²⁹ See Jessup 2008 for how motion can be part of Hobbes’s First Philosophy.

in the transition to Part IV of *De Corpore* (“Physics”). As the *conatus* of an inanimate body is its minutest directional motion, the *conatus* of an animate body is an *appetite* when it is in the direction of an object that promotes the life-sustaining vital motion of the blood, and *aversion* when it is away from an object that inhibits vital motion. Appetite and aversion are still *conati*, but their targets and effects are more specific than *conati* in general. In *Leviathan*, appetites and aversions form the basis of Hobbes’s psychology, thus also enabling the transition to statecraft, which is yet further down the subalternation chain.³¹

This approach has the promise of holding on to what is insightful in M&S’s approach and addressing the problem of order. As M&S emphasize, Hobbes’s terms do have the same *core* meaning in every domain, and so principles concerning, say, *conatus*, apply in exactly the same way to natural philosophy and politics, or inanimate matter and living things. However, each domain adds additional conditions that are peculiar to that domain: the endeavor of an animate body is not *identical* to the endeavor of an inanimate one, although each is an endeavor. The endeavor of an animate body is directed toward promotion of vital motion, motion that does not exist in the inanimate world. Yet both sustain the body undergoing them, as they are able. The problem of order is solved because the earlier, higher domains provide a more general account of the concept in question, which the lower sciences further specify. And so, the adding of conditions provides an appropriate criterion for ordering. This structure answers to the other points made about mixed mathematics in §4. A higher science provides the principle, say, that *conatus* is minute directional motion, which lower sciences take for granted. A discussion of *appetite* in the context of psychology no longer needs to include general deliberation about the nature of motion, since the properties that belong to the genus have already been determined. This provides the disciplinary insulation characteristic of mixed mathematics. Given Hobbes’s emphasis on causal definitions, the higher sciences also provide *the reason why* certain claims are true in the lower sciences. They specify causes.

6. CONCLUSION

Before closing, I need to address a historical problem. If subalternation was part of the lingua franca of late scholasticism, why should we think Hobbes fixed on it as an organizing disciplinary principle by thinking about *mixed-*

³⁰ Hobbes also adds that *conatus* must be towards something in *De Corpore* 15.5, OL I 182.

³¹ Strictly speaking, subalternation is a binary, asymmetrical, and transitive relation, so that if science C is subalternated to B and B is subalternated to A, C is subalternated to A. This means that sciences “lower” in Hobbes’s *Elements* are subalternated to all the sciences above them, up to first philosophy.

mathematics, and not by reading one of the many available scholastic treatments of the unity and structure of the sciences? My answer has two components. First, the question is somewhat misleading. Even within scholastic treatises and textbooks, the exemplars for subalternation were the mixed-mathematical sciences. I know of no discussion of subalternation that does not at some point mention the mixed-mathematical sciences as clear instances of the concept under discussion. With regard to disciplinary classification (and unlike the case of substantial forms, for example), being impressed with mixed mathematics did not stand opposed to taking scholastic doctrines to heart. Hobbes could have very well learned about the disciplinary and demonstrative status of mixed mathematics from sources that did not set out to champion the new sciences.

There is also another answer. Hobbes began thinking of a system of philosophy not during or shortly after his studies, nor while in Bacon's employ, nor while translating Thucydides, but while in and out of Paris, precisely when he was immersed in the culture of mixed mathematics that permeated Mersenne's circle. For Mersenne and his circle, however, championing mixed mathematics was politically plausible because of the science's curious disciplinary status: by virtue of their rank in the disciplinary hierarchy, the new mixed sciences did not challenge metaphysical and theological authority.³² A subalternating science, recall, was thought to be insulated from the sciences it subalternated. And so, if optical demonstrations could not cast doubt on geometrical proofs, they could certainly not cast doubt on the higher truths of theology and metaphysics. It was this feature of the mixed-mathematical sciences that rendered them safe: by practicing mixed-mathematics in the 1630s, Mersenne and friends were not only not challenging metaphysical and theological doctrine, orthodoxy suggested that they could not possibly do so! The Parisian Hobbes would have thus been uniquely positioned to think about the way in which demonstrations and disciplinary boundaries come together in the variety of geometrical sciences. Emulating geometry went hand in hand with avoiding "offensive novelty," as the epistle dedicatory to *De Corpore* claims to do.

I close by returning to the mathematization of early-modern philosophy. A good deal of literature is guided by a dual vision of early-modern mathematization efforts. On the one side stand Galileo, Huygens, and Newton, mixed mathematicians who sought piecemeal solutions to particular problems and who happily dispensed with essences and ultimate causes. On the other side stand Descartes, Hobbes, and Spinoza, philosophers who sought to build complete, architectonic systems that grounded all that there is in its

³² See, e.g., Garber 2002, 2004.

metaphysical roots. I think focusing on the demonstrative and disciplinary status of mixed mathematics suggests that, at least in Hobbes's case, the two were not as opposed as they now seem. In fact, the very structure of some architectonic systems might have been suggested by the mixed-mathematical approach. To put it tendentiously, some philosophers might have learned to philosophize from mixed mathematics.³³

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